

On the Bridging Powers of Geometry In the Study of Ancient Theatre Architecture

Zeynep Aktüre
Department of Architecture
Izmir Institute of Technology
Gülbahçe Campus, Urla
35430 Izmir, Turkey
E-mail: zeynepaktüre@iyte.edu.tr

Abstract

The on-going popularity of the Vitruvian layout for the Latin theatre is largely due to its capacity to bridge across several disciplines, which seems to appeal to a certain conception of material culture that assumes the existence of a plurality of formally similar structures of culture beyond surface phenomena. These tend to be not merely potent in their explanatory force but also gratifying aesthetically and ethically. Modern scholarship has forcefully promoted such a conjunction of truth, beauty, and goodness in the link between the Theatre in the Asklepieion at *Epidauros* and Pythagorean speculation. However, similar cognitively-significant structural or formal bridges would seem difficult to establish in all examples. In their absence, the search for a perfect geometry of perfect shapes beyond the extant remains may turn into a purely formalist exercise made possible by the capability of geometry to serve as an analytical tool through a reduction of the architectural code to a geometric code. This is a dilemma intrinsic in the difficult relation between architecture and geometry. In fact, Vitruvius seems to have noticed the problem long ago and tried to build a material bridge between his geometric assembly and the architectural project by recognizing the necessity to give up symmetry in the latter, wherever required by the nature of the site or the size of the project.

1. A Structural Bridge: The Vitruvian Layout for the Latin Theatre

David B. Small is one of the scholars who examined archaeological evidence for Roman period theatres to see that the majority were not constructed following the method described by the first century BC Roman architect Marcus Vitruvius Polio in the fifth book of his famous *De architectura*, which was dedicated to the Emperor Augustus [1]. This conclusion would appear to be in line with the critical opinion that Vitruvius' theory of architecture bears almost no relation to the realities of contemporary practice and had little influence on the near future [2]. His investigations led Small to devising, on the basis of the Vitruvian method, an alternative with greater explanatory power. His was one in a series of attempts including those of P.C. Hammond, F.B. Sear, Salvador Lara Ortega, and Luis Moranta Jaume [3, 4, 5, 6]. When taken together, these efforts attest the on-going popularity of the Vitruvian layout for the Latin theatre by taking it as a starting point, as do the eagerness of some archaeologists to adjust their evidence following Vitruvius' step-by-step method of design [1]. Examples for the latter would come from a wide geography, expanding from *Bilbilis Augusta* and *Segóbriga* in the Iberian to *Ancyra* in the Anatolian peninsulas and beyond [5, 6, 7, 8].

This paper will attempt to explain this on-going popularity of the Vitruvian layout by its capacity to bridge across several disciplines. To start with geometry, Vitruvius identifies the Latin theatre in the mirror of the Greek one, by the “*difference, that theatres designed from squares are meant to be used by Greeks, while Roman theatres are designed from equilateral triangles.*” [9] However, both design processes start with a circle inside which are inscribed, at equal distances apart and touching the boundary line of the circle, three squares in the Greek and four equilateral triangles in the Latin practice to produce

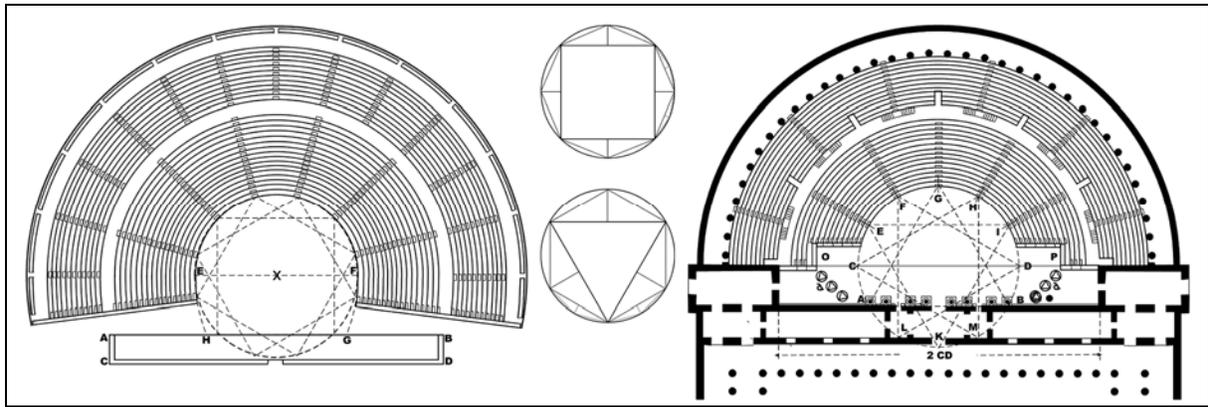


Figure 1: *The Vitruvian layouts for Greek (left) and the Latin theatres (right) and the quadrature of the circle according to Simplicius (centre top) and Themistius (centre bottom) [after 9 and 10].*

a circle of twelve points in both cases. The position of all the architectural components of both types is defined in the plan layout with respect to these initially-inscribed squares or triangles, in structural relation with each other. As to the design of the ascending rows of seats in the third dimension, Vitruvius argues for the convenience of the same dodecagonal layout for an arrangement with bronze sounding vessels that would satisfy perfectly the necessities of a physical phenomenon such as sound, on the basis of “*the canonical theory of the mathematicians and that of the musicians*”.

Modern scholarship has demonstrated that the equivalence of the two resulting schemes in terms of mathematical logic would suggest an origin in the two apparently different but essentially analogous methods used by the Sophists to resolve the quadrature of the circle [10]. Their overlapping would provide an embryonic image of the *signifer circulus* formula for celestial harmony, which was developed on the basis of the zodiac circle. As explained by the Geminus of Rhodes (110-40 BC), it was the position of the signs in trigons on the sphere of the fixed and of those in quadrature that produced an accord with their opposition. The circle or the sphere were, by definition, generators of perfection and the inscription of a figure inside a circle or a solid inside a sphere was the proof, both for the ancient geometrician and for the ancient philosopher, of the accomplished character of that figure or object. The impossibility of inscribing any other perfect shape into a circle of twelve zodiacal points would suggest the idea of an innate binarism for the two Vitruvian schemes. This idea of binarism, appearing within the context of a pair of geometries that apparently bridges, as such, across the disciplines of geometry, architecture, physics, mathematics, music, logic, astronomy, philosophy, and taxonomy, would lead to a certain conception of material culture whose critical evaluation may help in a better understanding as to why such congruence would appear to provide a more satisfactory explanation of architectural phenomena than any other that would evaluate them as objects in and of themselves.

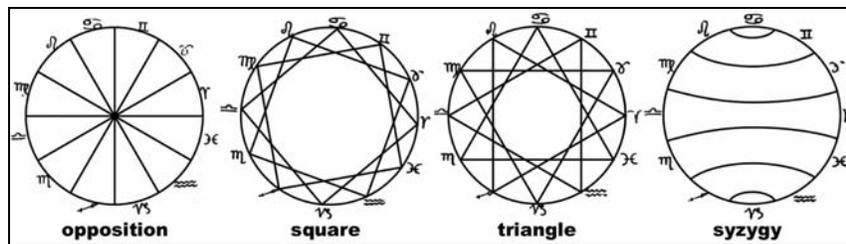


Figure 2: *The signifer circulus formula as explained by the Geminus of Rhodes [after 10].*

The reference here is to a model of *structuralisme* constructed by Ernest Gellner as a generative method consisting of a set of ideas or themes from which arguably follow the observed activities and positions of people normally described as structuralists [11]. These include a belief in the existence of a plurality of structures that “*formally resemble each other and are rooted in some generically shared structure of the human mind.*” In the analyses of material culture, the idea of a core set of elements (or structure), that are generally assumed to occur in a pair of polar opposites and constitute the limits of the world in question, comes to the fore as generating the system of signs. This would accord with Gellner’s classification of structuralism as a form of emanation, which is one of the two main conceptions of causation, with the other being covering law. The two are parallel in their mutual presumption of a permanent core or another reality beyond surface phenomena. Emanationist explanations assume that the regularities discerned in the phenomena open to view emanate or flow from that permanent core that is normally (or permanently) hidden from view. Any attempt to bind phenomenal regularities under generalizations, including surface classification and prediction, is bound to remain superficial without an understanding of those inner forms that are, on the other hand, accessible only through their alleged manifestations in the phenomena. “*Those inner forms tend to be not merely potent in their explanatory force but also gratifying aesthetically and ethically. They reveal a moral as well as an ontological order; in fact these various orders converge. Truth, beauty, and goodness are one.*” [11] In this way, emanationist explanations provide, not only shorthand summaries of surface patterns, but also “*a deep, permanent, morally saturated, and satisfying reality, qualitatively different from and superior to the ephemeral and amoral connections observed on the surface of things.*” [11] The on-going appeal of the Vitruvian layout for the Latin theatre seems to be rooted in its capacity to reveal such a reality, in the face of (or perhaps due to) the fact that the corpus of ancient theatre remains is too amorphous to be equally gratifying in itself.

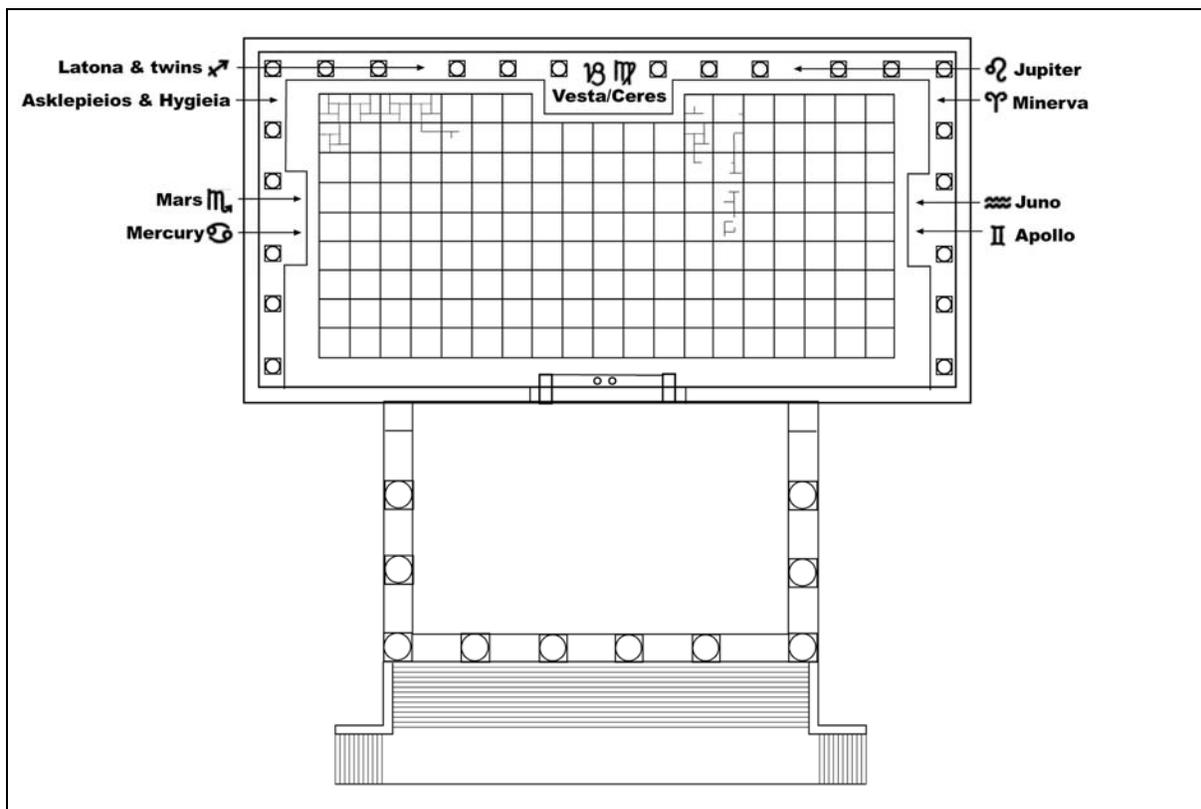


Figure 3: The reconstruction of the statuary program at the aedes Concordiae Augustae [after 13].

Architecture is a phenomenon beyond which the existence of another reality is, not presumed, but known to exist: the conscious will and performance of man. Beyond the Vitruvian layout for the Latin theatre, with all its cross-disciplinary associations, there would seem to exist the conscious will and performance of one man: the Emperor Augustus, who is portrayed by Paul Zanker as having structured an Imperial Roman culture using the power of images [12]. So, Pierre Gros would seem to have a strong point in suggesting that the *signifer circulus* formula for celestial harmony may have had a more direct application on the Vitruvian scheme for the Latin theatre than generally admitted [10]. A most direct reference is to be found in the sculptural program of the Temple of Concordia Augusta in Rome wherein the pairings of Juno and Apollo, Mars and Mercury, and Vesta and Ceres would have served to represent the harmonious world order that is being established by Augustus through the very principle of opposition in trigons and quadrature on the sphere of the fixed, as mentioned by the Geminos of Rhodes: “*Vesta rules Capricorn, Ceres, and Virgo, and Capricorn and Virgo are in the same trigon; Mars rules Scorpio, Mercury, and Cancer, and Scorpio and Cancer are members of another trigon; Juno rules Aquarius, Apollo, and Gemini, and Aquarius and Gemini are members of the third trigon. Significantly, Manilius points out that relations within a trigon were not always untroubled, but ultimately harmony prevailed and balance was maintained.*” [13] Vitruvius’ dedication of a whole book to the Latin theatre would have indicated his awareness of the key role to be played by the theatrical edifice in the establishment of this difficult balance within the new world emerging under Augustus.

2. A Formal Bridge: The Theatre of the Asklepieion at *Epidaurus*

As such, the Vitruvian layout for the Latin theatre would seem to owe its bridging powers to a careful structuring by the Roman architect of the current ideas of his time to serve the political agenda of the first Roman emperor to whom his treatise on architecture was dedicated. This oldest surviving treatise on architecture was rediscovered and printed for the first time in 1484 [14]. Publications documenting attempts at a restitution of ancient theatre remains in the Iberian Peninsula following Vitruvius’ directions date back to as early as the sixteenth century, possibly due to the influence of the *Medidas del Romano* written by Diego de Sagredo, which appeared in Spanish in 1526 [15]. From that time on, the Vitruvian model has been so influential on our modern views about the Greek and Roman theatres that it served as a measure for architectural achievement in ancient theatres. A notable example in this respect is Gottfried Gruben’s description of the renowned Hellenistic Theatre of the Asklepieion at *Epidaurus*, on the basis of Wilhelm Dörpfeld’s early plan of 1896, as the best among all surviving theatres in displaying the geometrically determined principles of design referred to by Vitruvius that manifested, according to the author, in the division of its *cavea* into twelve by taking as reference the corners of a dodecagon inscribed in the basic circle of the *orchestra* [16].

However, very detailed and precise measurements taken by Armin von Gerkan and Wolfgang Müller-Wiener in the period 1953-56 were soon to establish the basic organizing scheme possibly of the whole edifice to be the inner circle of its *orchestra* inscribed by a perfect pentagon [17]. Associated traditionally with Venus, the pentagon was apparently more potent in its explanatory force than the square or the equilateral triangle for the case of *Epidaurus*, as it described a five-pointed star that symbolized *Hygieia*, daughter of Asklepieios and personification of health, for the Pythagoreans [18]. Around 450 BC, Greek thinkers had discovered the Golden Ratio from Pythagorean speculation and from the inscription of this very form inside a circle. There are echoes of this theorem in Plato’s *Timaeus* (31b-33c), in the Treasury of *Cyrene* at *Delphi* designed as a demonstration and exemplification of mathematical wisdom, and in literary masterpieces including works of Vergil, Lucretius, Catullus, and Lucan, wherein passages are divided into line-totals that are in ratio to each other and to the whole passage or poem in Golden Section relations [19]. Jean Bousquet has illustrated the use of the Golden Section, and the Fibonacci Series intimately related to it, in the arrangement of the tiers in the *cavea* of the Theatre of *Epidaurus* [20]. There are 21 rows in the upper and 34 in the lower *cavea*, which sum up to 55. All three

are numbers in the Fibonacci Series and 21:34 is in ratio to 34:55. Additionally, these numbers constitute a Grand Tetractys. R.V. Schoder believes that, the architect's choice for precisely these numbers would have resulted from "*a deliberate desire to work into his design the perfections and mysterious interrelations of these numbers which contemporary Pythagorean speculation extolled.*" [19] So, here again, we seem to be faced with a layout that owes its bridging powers to a careful structuring of the current ideas of a certain period but this time the product is a work of architecture, instead of a verbal description of a design method in a treatise on architecture. This would seem to make a great difference with Schoder's evaluation of it as "*the most aesthetically pleasing arrangement in the history of theatre design*" [21]:

True, an observer is not quite aware of all these mathematical correlations in precise equivalents and factors and multiples, but he is aware of the over-all harmonious symmetry, and pleased by it. [...] As in understanding a painting, so here one discovers the details and complex associations of individual elements only by close-up study and analysis, which seems mechanical and dehumanizing; but this necessary process of factual dissection and part-by-part examination leads to a better aesthetic appreciation also and is the sole means of discovering and adverting to—and thereby enjoying—some facets of the artistic accomplishment. [16]

With this statement, we seem to move one step closer to the structuralist idea of "*some generically shared structure of the human mind*" that is capable of appreciating the beauty of a work of art or architecture through an understanding of the underlying geometric forms and mathematical correlations. In this way, we also come closer to what corresponds to aesthetic formalism in the philosophy art. Aesthetic formalism may be described as a body of ideas according to which the aesthetic value of natural or manmade objects is determined by their form understood as opposed to matter, as opposed to content, as opposed to formlessness or, notably, as structure where the structure is constituted by a system of relations [22]. Theoreticians considered to be formalists have used the notion of form primarily to denote "*a certain arrangement of parts, a structure of elements, or a global composition of elements of a work or some other object*" and, therefore, in opposition to matter or substance [23]. At the origin of artistic formalism was a protest against the treatment of works of art as substitutes for politics, morality, or religion, as in Victorian expectations for an enhancement in the morality of the society through literature. This defense of art's autonomy by renowned theoreticians and critics working in diverse fields has had its share in the notion of the uniqueness of individual art forms and values exclusive to each one of them. However, this notion has had the risk of leading to an experience and appreciation works of art exclusively as objects in and of themselves rather than within a broader artistic and cultural context, without an acknowledgement of the cultural nature of art or the cognitive significance of individual works beyond their purely aesthetical properties. This has been marked as a major weakness of formalist approaches, together with their strong normativistic tendencies that reveal in an imposition of aesthetic experiences based on formal aspects as the only correct model of art reception and art evaluation. [23]

Reviewing the extant literature on the Theatre of *Epidauros* under the light of these criticisms would reveal the determining influence of the artistic and cultural context of the monument, as well as the cognitive significance of the design tools used in the shaping of its individual components over the resulting aesthetical experience. The building is thought to be part of an ambitious building program implemented during the fourth and third centuries BC in the Sanctuary of Asklepios that had become, by that time, the most celebrated healing centre of the ancient world [24]. A Polykleitos the Younger was employed for the design of the monument, who must have been an Argolid architect rather than the renowned fifth century sculptor [19]. Nevertheless, he is one of the very few ancient architects whose name has survived up to our times in connection with a theatre design, perhaps due to the significance of that very design itself. In the light of the previously reported analyses, Polykleitos would seem to have come up with an idea that is meaningful in (and probably only in) connection with the cult of Asklepios although the very same scheme would have been equally gratifying aesthetically in another context if it were to be judged "*only by a close-up study and analysis*", as suggested by Schoder, and not as part of a unique natural and cultural landscape. It apparently was the prosperity of the healing complex inserted in that landscape that made possible the application of that design idea in such precision that modern scholars could reveal

the geometric and mathematical relations structuring it by working back from the extant remains and then discover the associations of those relations with Pythagorean speculation. Their analyses have provided a potent and gratifying explanation of the impressive architectural characteristics of the edifice.

3. As a Conclusion: A Material Bridge

The mechanism here seems to be the one described by Umberto Eco through a distinction between the denotations and connotations of architectural objects [25]. While the denoted meaning of architectural objects is the function they make possible, their formal characteristics may also “refer to a certain conception of inhabitation and use; they may connote an overall ideology that has informed the architect’s operation.” In the semiotic mechanism described by Eco, connotations of architectural objects rest on the denotation of their primary function. In an attempt to move back from the complex and temporal connotations, it might be tempting “to hypothesize for architecture something like the ‘double articulation’ found in verbal languages, and assume that the most basic level of articulation (that is, the units constituting the ‘second’ articulation) would be a matter of geometry.” Accordingly, the Euclidean *stoicheia* (i.e. the elements of classical geometry such as the point, the angle, or various curves and the straight line) would combine into certain high-level spatial units called *choremes* (i.e. the square, the triangle, the parallelogram, the ellipse, and even rather complicated irregular figures that can be defined with geometric equations of some kinds) that belong to a “first articulation” and begin to be significant. The “second articulation” would be based on these units. A third level of articulation is suggested by solid geometry and other articulation possibilities may be assumed with non-Euclidean geometries.

The same code may be argued to lay behind some artistic phenomena, including abstract and representational art, on the premise of the long-held conviction that configurations in the latter can be reduced to a rather complex articulation of primordial geometric elements. In this way, the analytic possibilities offered by geometry enable a comparison of architectural phenomena with other type of phenomena by describing them in the same terms, as in the case of the Vitruvian schemes or the analyses of the Theatre of *Epidauros*. For Eco, this reveals the capability of geometry to serve as a *metalanguage* that might even be identified with a *gestaltic* code presiding over our perception of all such forms, as assumed by *structuralistes*. The problem is that, “the fact that architecture can be described in terms of geometry does not indicate that architecture as such is founded on a geometric code.” [25] An analogous example comes from language studies wherein the fact that both Chinese and Italian words are articulated in phonemes and, therefore, can be seen as a matter of amplitudes, frequencies, or wave forms does not indicate that “Chinese and Italian rest on one and the same code; it simply shows that the languages admit of that type of analysis, that for certain purposes they can be reduced to a common system of transcription.” [25] This would reveal a dilemma intrinsic in the relation between architecture and geometry in analytical studies on ancient theatre architecture, which is rooted in the fact that all ancient theatre buildings would have been laid out using a geometric assembly one some sort. Gros observes that Vitruvius had noticed the problem long ago and tried to establish a more direct relation between his geometric assembly and the architectural project with recommendations for giving up symmetry for the sake of utility in steps, curved cross-aisles, their parapets, the passages, stairways, stages, tribunals and the like, which need to have the same size both in a small and a large theatre [9, 10]. In the same paragraph, Vitruvius admits the impossibility of applying the proportional systems he proposes in all theatre constructions. “Instead, it is up to the architect to note in which dimensions it will be necessary to pursue symmetry and in which to make adjustments according to the nature of the site or the size of the project.” [9] And it is up to us, the modern investigators of ancient theatres, to decide in which theatres it would be meaningful to search for a perfect geometry of perfect shapes beyond the extant remains and in which to be contended with whatever remains we have in hand, without attempting to move back from them with the hope of finding a more satisfactory explanation in the connotations of such geometries.

It would seem that certain buildings such as the Theatre of *Epidauros*, with its highly symbolic religious connotations within the Sanctuary of Asklepios, would afford all the purposes required for such a reduction to a common system of transcription through a description in terms of geometry. Others that are located in major administrative centers of the Roman Empire such as *Ancyra*, the capital of the Roman province of *Galatia* in Anatolia, may motivate an analyses in reference to the Vitruvian layout for the Latin theatre, if by nothing else, by the direct link established with the Augustan project through the dominating presence of the nearby Temple of Augustus and Roma that features the monumental inscription panels of the complete text, in Latin and in Greek translation, of *res gestae divi Augusti*, the first and last report given by a Roman emperor, the Emperor Augustus, to his nation of his life and deeds. However, not all ancient theatres, Greek or Roman, may be expected to have equally strong connotations. Some, such as the Theatre of *Segóbriga*, were apparently constructed only to perform their denoted function and with a limited budget that could be allocated so late as the aftermath of Vespasian's grant of municipal status (*ius Latii*) to communities in the province of *Baetica* in Iberia. This should not be taken to mean the impossibility of their having been designed after an elaborate system of geometrical or mathematical relations beyond their material remains. Lara Ortega and Moranta Jaume have separately suggested two schemes for the monument featuring respectively four and three equilateral triangles inscribed in a circle, taking as a basis the Vitruvian layout for the Latin theatre [5, 6]. It should be taken, instead, as a warning against the irresistible charms of fabricating one such layout for the more deteriorated examples, such as the Theatre of *Bilbilis Augusta* [7].

References

- [1] D.B. Small, 'Studies in Roman Theater Design', *American Journal of Archaeology* 87 (1983) pp.55-68.
- [2] John Onions, *Bearers of Meaning - The Classical Orders in Antiquity, the Middle Ages, and the Renaissance* (Princeton, New Jersey: Princeton University Press, 1988) pp.40-41.
- [3] P.C. Hammond, *The Excavation of the Main Theatre at Petra, 1961-1962 - Final Report* (London: Colt Archaeological Institute Publications, Bernard Quaritch Ltd., 1965).
- [4] F.B. Sear, 'Vitruvius and Roman Theater Design', *American Journal of Archaeology* 94 (1990) pp.249-58.
- [5] Salvador Lara Ortega, 'El trazado vitrubiano como mecanismo abierto de implantación y ampliación de los teatros romanos', *Archivo Español de Arqueología* 65 (1992) pp.151-79.
- [6] Luis Moranta Jaume, *Hipótesis de la Existencia de un Teatro Romano en Palma de Mallorca* (<http://palma.infotelecom.es/~moranta>, accessed on September 22, 2000).
- [7] Martín Martín-Bueno and J. Núñez Marcén 'El teatro Municipium Augusta Bilbilis' in *Teatros Romanos de Hispania*, edited by S. Ramallo and F. Santiuste (Murcia: Universidad de Murcia y Colegio de Arquitectos de Murcia, 1993) pp. 119-32.
- [8] Musa Kadioğlu, 'Ankyra Tiyatrosu: Ön Rapor' in *I.-II. Ulusal Arkeolojik Araştırmalar Sempozyumu*, edited by Tunç Sipahi and Zeynep Çizmeli Öğün (Ankara: AÜDTCF Yayınları, 2004) pp.123-40.
- [9] Vitruvius, *The Ten Books on Architecture*, translated by Morris Hicky Morgan (New York: Dover Publications, Inc., 1960; reprint of the 1914 Harvard University Press edition) p.153.
- [10] Pierre Gros, 'Le schème Vitruvien du théâtre latin et sa signification dans le système normatif du *De Architectura*', *Revue Archéologique* (1/1994) pp.57-80.

- [11] Ernest Gellner, 'What is structuralisme?', in *Theory and Explanation in Archaeology*, edited by Colin Renfrew, Michael Rowlands and B.A. Segraves. (New York: Academic Press, 1982), pp.97-124.
- [12] Paul Zanker, *The Power of Images in the Age of Augustus*, translated into English by Alan Shapiro (Michigan, Ann Arbor: The University of Michigan Press, 1990).
- [13] B.A. Kellum, 'The City Adorned: Programmatic Display at the *Aedes Concordiae Augustae*' in *Between Republic and Empire – Interpretations of Augustus and His Principate*, edited by Kurt A. Raaflaub and Mark Toher (Berkeley, Los Angeles & Oxford: University of California Press, 1990) p.295.
- [14] Margarete Bieber *The History of the Greek and Roman Theater* (Princeton, New Jersey: Princeton University Press, 1961) p.255.
- [15] Joseph Rykwert *The Dancing Column - On Order in Architecture* (Cambridge, Massachusetts; London, England: The MIT Press, 1996) p.401.
- [16] Helmut Berve and Gottfried Gruben, *The Architecture of the Temples, Theatres and Shrines*, volume 2 (London: Thames and Hudson, 1963).
- [17] Armin von Gerkan and Wolfgang Müller-Wiener, *Das Theater von Epidauros* (Stuttgart: W. Kohlhammer, 1961).
- [18] Lutz Käppel, 'Das Theater von Epidauros. Die mathematische Grundidee des Gesamtwurfs und ihr möglicher Sinn', *Jahrbuch des Deutschen Archäologischen Instituts* 104 (1989) pp.83-106 cited in J.R. Green, 'Theatre Production: 1987-1995', *Lustrum* 37 (1995 [1998]) p.52.
- [19] R.V. Schoder, 'II. The Theatre at Epidauros as a Work of Art', in *Greek Drama – A Collection of Festival Papers II*, edited by Grace Lucile Beede (Vermillion, South Dakota: The Dakota Press, the University of South Dakota, 1967) pp.13-39.
- [20] Jean Bousquet, 'Harmonie au Théâtre d'Epidaure', *Revue Archéologique* 41 (1953) pp.41-49.
- [21] R.V. Schoder *Ancient Greece from the Air* (London: Thames and Hudson, 1974) p.66.
- [22] David Pole, *Aesthetic, Form and Emotion* (London: Duckworth, 1983) cited in [23].
- [23] Bohdan Dzemiđok, 'Artistic formalism: its achievements and weaknesses', *The Journal of Aesthetics and Art Criticism* 51/2 (1993) pp.185-193.
- [24] Juan Miguel Casillas and César Fornis 1995. 'Epidauro y el culto a Asclepio', *Revista de Arqueología* 173 (1995) pp.28-39.
- [25] Umberto Eco, 'Function and Sign: The Semiotics of Architecture', in *Rethinking Architecture – A Reader in Cultural Theory*, edited by Neil Leach (London and New York: Routledge, 1997) pp.182-202.