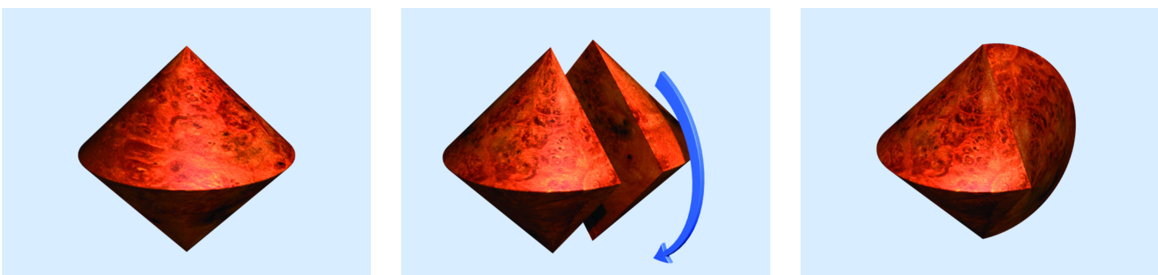
Streptohedrons (Twisted polygons)

David Springett
8, Strath Close
Rugby, Warks, England, CV21 4GA
E-mail: david@cdspringett.fsnet.co.uk
website: davidspringett.fws1.com

Abstract

Imagine a simple form, a cone with a symmetrical cross-section. Now split that cone from apex to base, twist the two halves and re-join. Before your eyes a new, complex form is produced. Imagine more intricate geometric solids which are split, twisted and re-joined, magically producing shapes which coil and twirl - shapes not seen before, unexplored shapes. Remove the inner form of some of these twisted shapes and a path or ribbon remains. These shapes, these ribbons, this idea, will excite the Mathematician, the Sculptor and artist alike.

In June 2001 I was shown two intriguing shapes. The first was in the form of two cones, fixed base to base, which had been split from apex to apex. The two halves had then been separated, one half rotated 90° before being glued to create a form with two edges and one side. (See figures 1, 2 and 3). I later discovered that this double, twisted, cone was developed by Colin Roberts in 1967 and was called a Sphericon. [1]



Figures 1, 2, and 3, creating a Sphericon

The second shape was a cone, its cross section being an equilateral triangle, which had been split vertically from apex to the centre of the base. The two halves had then been rotated so that the edge of one half lay against the baseline all the other half. The two halves had been glued together produce a form which had one side and one edge. (See figure 4).

This twisted double cone rolls drunkenly down a slope, the twisted single cone also rolls in a bizarre manner but stops on one of its half bases. For some reason I also remembered a third similar shape, similar in that it was also a solid of revolution which had been spit along axis twisted and rejoined. This was in the form of a cylinder with a wide disc at its centre. (Figures 5 and 6). It was then that I looked more carefully at the cross-section of these three solids of revolution. The first, the Sphericon, I had mistakenly viewed as a diamond shape. (Third form the right, top line of figure. 7 and photo. 1 second row, extreme right). I should have seen it as a square with the centre of rotation through its points. The second has an equilateral triangle as its cross-section. The third has a cross-shaped cross-section. (Figure 8). I put aside the third shape, for the moment, for it did not yet fit in with the pattern I had just seen.

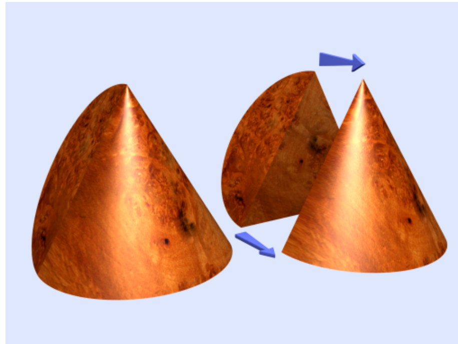
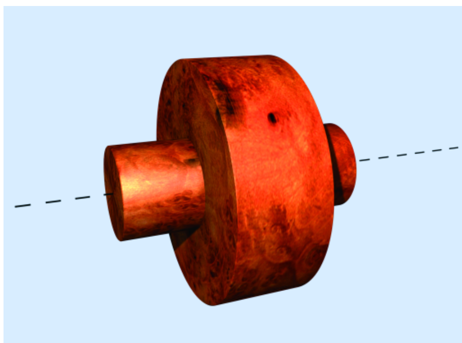
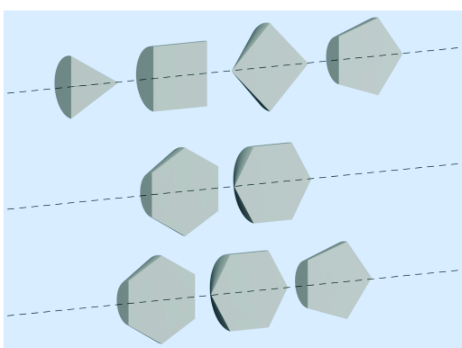
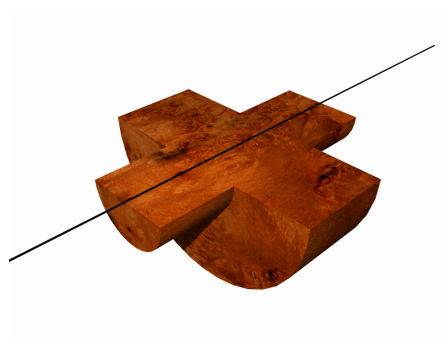


Figure 4, rotated cone



Figures 5 and 6

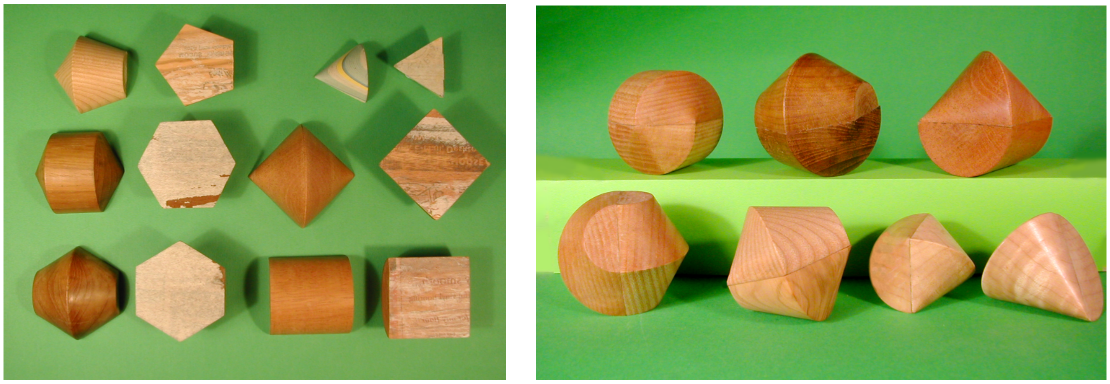




Figures 7 and 8

A triangular cross-section, a square cross-section, obviously the next shape to work with would be a Pentagon. A regular Pentagon can be used as the base for a solid of revolution, the axis will be through the apex then crossing the midpoint of the base. (Top row, extreme right, figure 7).

When I split, twisted and rejoined this piece I discovered two very interesting details. First it produced two forms. Move the apex of one half to join the rim of the of the other half for shape number one. Move the apex of one half to join the base of the other half for shape number two. A second detail I discovered was that the shape could have a right-hand or left-hand twist. (Photo. 2 bottom row, second from left).



Photos 1 and 2

A triangle, a square, a Pentagon, it was natural to move on to produce a solid of revolution based on a regular hexagon. So with the axis lined up through the flat of the base, through its centre and the flat of the top I produced an hexagonal form. Only a single form could be made by splitting, twisting and rejoining, but again it can have a left or a right-handed twist. (Figure 9 and photo.2 bottom line, first left).

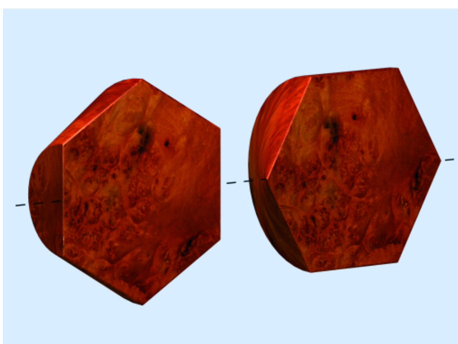


Figure 9 hexagonal shape

Photo 3 right and left handed forms

Looking again I realized that a hexagon has two different centres of rotation, one through its flats as I had produced above and a second through its points. I produced this second hexagonal shape, with the centre of rotation through its points, and discovered a second hexagonal twisted form. Again this could have a left or right-handed twist. (Fig. 9, photo.2 second left).

I was surprised when I looked at the two hexagonal forms which were lying on the table in in their split halves, each had the same size hexagonal base shape. I took one of the halves which had been rotated through the points and joined it to one of the halves which had been rotated through the flats to produce a hybrid. This hybrid could be right or left-handed. (Photo. 3). Having realized that polygons with equal number of sides can have two centres of revolution I returned to the Sphericon. It has a square as its base form and its centre of revolution is through the points but if the square is rotated through its flats then a simple cylinder is produced.

A rather unexciting form is produced when the two halves of the cylinder are twisted and refitted (Photo. 4). Its outer edge follows the curve similar to the seam line on a tennis ball. A more interesting hybrid is formed using one half of the Sphericon and one half of the cylinder.

Figure 10 star pentagon

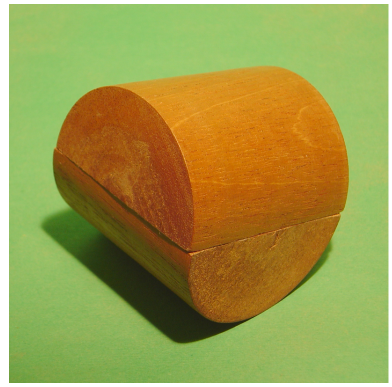


Photo 4 twisted cylinder

After a few days of thought it suddenly occurred to me that a regular star form could be used as the basis for further twisted solids of revolution. I first produced a regular pentagonal star drawn by joining points 1, 3, 5, 2 and 4 on a regular Pentagon. (Figure 10).

Remembering that the pentagonal solid, when split, twisted & re-joined, produced two forms, I twisted the top point of the star to meet with the edge. This produced form number one. (Photo. 5). Next I moved the top point of the star to meet with the base. This produced form number two. (Photo. 5). Of course there were left and right hand versions of both twisted forms.

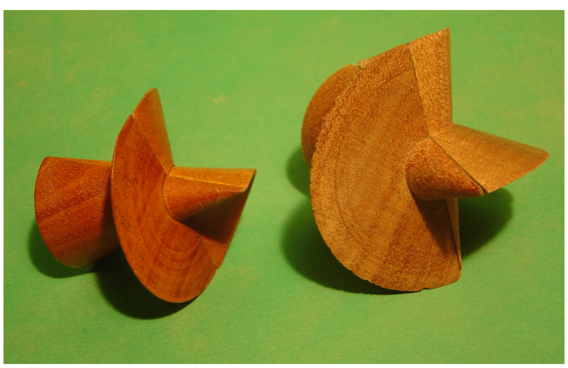


Photo 5 twisted start pentagon

Photo 6 other stars

A three pointed star followed, looking much like the Mercedes symbol. From this I was able to produce one form only but it can have left or right hand versions. (See figure 11 and photo 6 plus photo 7 bottom right). The four pointed star looks quite ordinary. It has two forms, one with the axis of revolution through its points, the other with the axis of revolution through the valleys. (Fig. 11 and photo 6). And, of course, the right and left-handed versions of each of these, added to which there is the hybrid version.

A six pointed star was more exciting producing a more "twisted" form. Again there are two axes of revolution, the first through the points and the second through the valleys and there are right and left hand version of each of these plus the hybrid. (Figure 11 and photo. 7 top right).

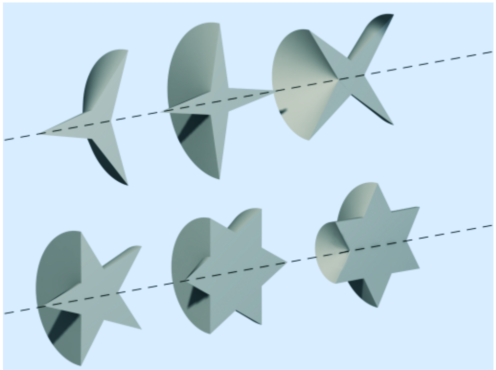


Figure 11 axes **Photo 7** models

At this point I returned to the cross shaped, cross section pieces (figure 6) trying to work out how this fitted in with the pieces I had produced so far. It became apparent that this form was similar to the star shapes but the arms would have parallel sides and ends rather than a point.

I produced a three armed version (photo. 8 top right & photo. 9 bottom right) which when split & twisted gave a very satisfying twisted form, either right or left-handed. The five arm piece was particularly interesting (two forms plus left and right handed) with a snaking valley twisting around the form. (Photo. 8 top left and photo. 9 top right). The six arm shape was produced with the axis running through the arms (left and right handed versions) or the axis running through the valleys (left and right handed versions) and of course the hybrid, all stunningly sinuous. (Photo 8 bottom left and photo. 9 top left).

Photo 8 various arm splits **Photo 9** models

I returned to the simple cross and turned it so that the axis of revolution was set through the valleys. (Photo. 8 bottom right). The resulting shape was unexpected and produced one form only (Photo.9, bottom left) but combined with half the original form (figure.8) it produced an interesting hybrid. (Photo. 10).

I have run an undercut groove into the corners of a hexagonally based shape allowing ball bearings to run in that groove. When the shape had been split the ball bearings were inserted into the groove, the halves twisted and re-glued producing an amusing piece with the ball bearings running around the twisted track. (Photo.11 middle top).

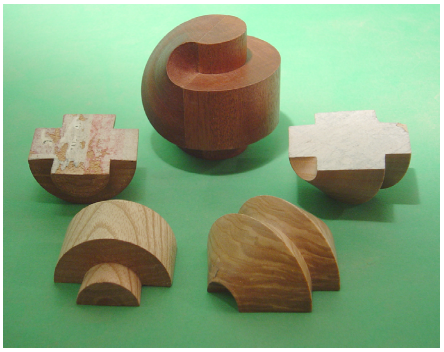


Photo 10

Photo 11

Properties

Before I explain the areas I wish to explore I will detail some of the properties of these pieces.

1. These Streptohedrons will fit inside a sphere (the size of that sphere relates to the base size of the polygon) and their extreme edges or points will touch the inside of that sphere whether they are in their left or right handed form, whether they are twisted or aligned, or whether they are in the hybrid form.
2. These forms are based on regular polygonal figures which have rotational symmetry.
3. These streptohedron can be produced in left or right-handed forms.
4. Those forms based on equal sided regular polygons (star and arm forms included) will produce hybrid forms.
5. Those forms based on odd sided regular polygons (star and arm forms included) will NOT have a hybrid form.
6. All these streptohedron forms roll in an unusual "drunken" manner similar to the "two disc rollers" or "wobblers".

Photo 12

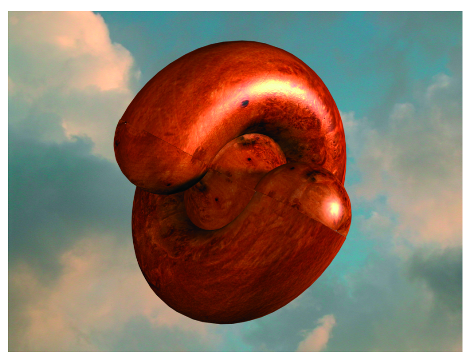


Figure 12

Areas to explore

I have begun to introduce curves into these shapes as can be seen in photo. 11 middle left. This piece is based on the cross form seen in figure 8 but it has concave curves cut into the faces. Figure 11 shows that same cross base with convex (almost spherical) ends. Also in photo. 11, middle bottom, it will be seen that curves have been worked into three of the six faces of an hexagonal form.

On the right, in photo. 12 is a figure which is based on a Reuleaux triangle which has constant diameter but not a fixed centre. On the left of photo. 12 can be seen a stepped form, based on an equilateral triangle, which looks like an Art Deco Ziggurat. I have not developed any of these final pieces using their different centres of revolution, produced hybrids or extended the curves more deeply.

At present I am producing some of these forms so that the centre is removed producing a more ribbon-like form. (Photos 13 and 14). The negative shapes within these "ribbons" are worth exploring. Photos 15 and 16 show a ribbon form based on an equilateral triangle. The negative centre is a twisted hexagon, precisely the same as shown in photo 2 bottom left.

Shapes and their relationships continue to present themselves. Returning to the beginning ... the double cone (figure 1 and photo 17). This is a solid of revolution based on a square. Now take the same size square and, using on corner as the axis rotate it to produce a ring as shown in split form in photo 18. Next fit double cones into the hollow ends of that ring. (Cross-section shown in photo 19). This produces a larger double cone (photo. 20). If this double cone is split, twisted and rejoined it makes a sphericon (photo 21). Pull this sphericon apart and two symmetrical halves are revealed (photo 22). I can only show the next stage in half-section at present. Apply another ring, (same size as in photo 18), to the end of the double cone filling the end hollow with a third double cone (photo 23). If a further, larger, ring (shown as white cross-section and wire in photo24) is applied a double cone of a greater size is made. This much larger double cone can be split, twisted and rejoined to make a sphericon or it may be pulled apart revealing symmetrical parts.

And finally, for the moment, that central shape looks familiar. It is the cross, cross-section figure seen in photo 10. The variety of the shapes I have uncovered seem endless and linked just like the pathways on some of these forms.

Photo 13

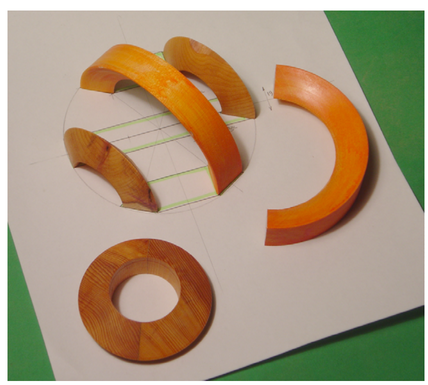
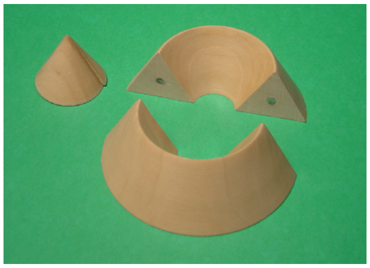
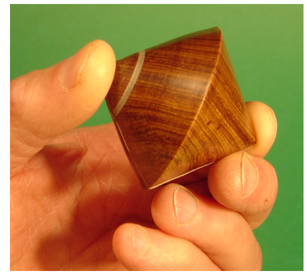


Photo 14

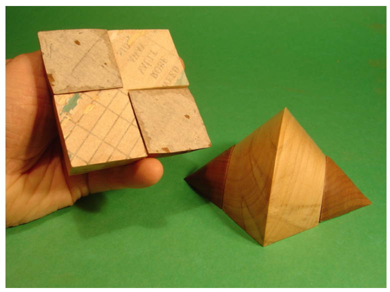
References

1. Ian Stewart, Scientific American October 1999
2. David Springett, “Streptohedrons (spheriforms)”, *Society of Ornamental Turners Bulletin* No 111 Sept. 2004
3. David Springett “Streptohedrons” a two part article *Woodturning Magazine* issue 135 Spring 2004 & issue 136 May 2004 (GMC publication)
4. David Springett, *Woodturning Wizardry*, 1993, reprinted and revised 2005, Guild of Master Craftsmen Publications Ltd

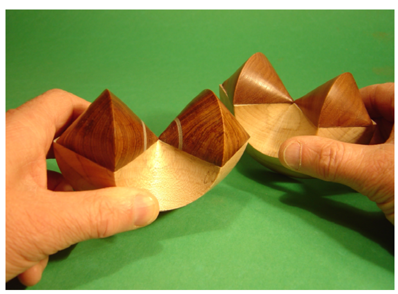
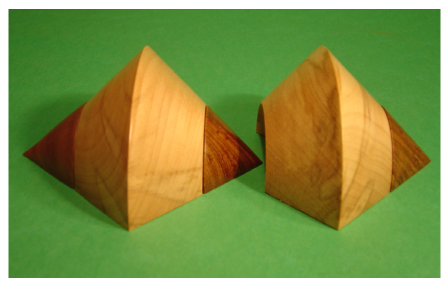




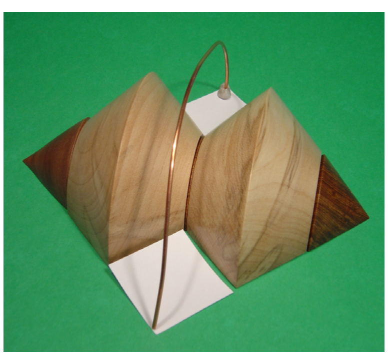
Photos 15, 16, and 17


Photos 18, 19 and 20

Photos 21, 22 and 23




Photos 24 and 25

Acknowledgements

The figures are by Robin Springett and first appeared in *Woodturning Magazine* issues 135 and 136 in spring 2004. Reproduced by permission of GMC Publications, Lewes, UK.