Coxeter and the Artists: two-way inspiration, Part 2

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Abstract

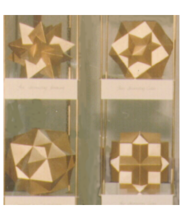
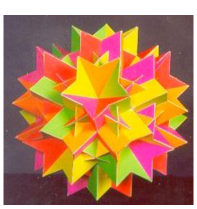
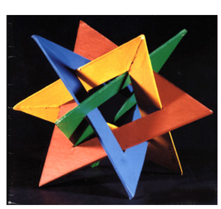
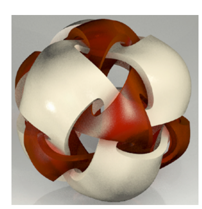
H.S.M. Coxeter's delight in geometric form was the catalyst for fruitful two-way interactions with artists, sculptors, and model-makers. His publications and personal encouragement inspired them to create breathtaking expressions of the power and beauty of geometry. The inspiration was often reciprocal: the artists' expressions, frequently guided by their unique intuition, sparked Coxeter's geometric curiosity and often provided new mathematical insight.

Introduction

The chief reason for studying regular polyhedra is still the same as in the time of the Pythagoreans, namely, that their symmetrical shapes appeal to one's artistic sense. —H.S.M. Coxeter [4, preface].

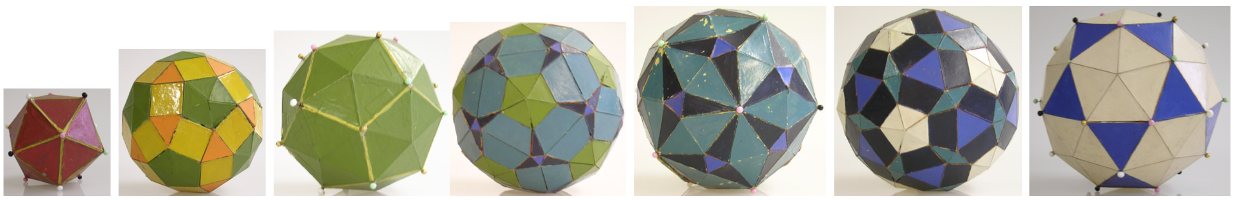
During his lifetime, H.S.M. (Donald) Coxeter interacted with many artists and model-makers who shared his passion for symmetric geometric form. Often it was one of his many publications on polyhedra that inspired the artists to create drawings, models of paper, card, wire, wood, or plastic, or to invent original sculptures whose beauty exploited the underlying geometry. Sometimes their work was done purely intuitively, and "discovered" by a mathematician who brought it to Coxeter's attention. When Coxeter learned (directly or indirectly) of the mathematically interesting work of an artist or model-maker, he never hesitated to reach out to the artisan, to share his own insights, and to ask his questions. He was very aware of the heuristic value of physical models in discovering new mathematical relationships, and in communicating mathematical information visually.

Several of the artists had no formal studies in mathematics beyond high school, others had some formal education in higher mathematics, and some held doctorates in mathematical fields. Coxeter eagerly sought to understand the often startling insight of those without formal training in mathematics, and gave credit in his publications to their discoveries. With mathematician-artists, there were often fruitful mathematical collaborations. Alicia Boole Stott (1860-1940), Paul Donchian (1895-1967), M.C. Escher (1898-1972), Magnus Wenninger (b. 1919), Michael Longuet-Higgins (b. 1925), A.G. (Tony) Bomford (1927-2003), John Robinson (b. 1935), George Odom (b. 1941), Peter McMullen (b. 1942), Rinus Roelofs (b. 1954), George Hart (b. 1955), and Marc Pelletier (b. 1959) represent the wide pantheon of artists and model-makers who share in Coxeter's legacy. I hope that by telling their stories, this part of Coxeter's life will become better known. Here, I tell of Coxeter's collaborations with Stott, Longuet-Higgins, McMullen, and Bomford. Detailed stories about the others are in my first article [26]; some of their art is shown below (left to right: by Donchian, Wenninger, Robinson, Odom, Hart, and Roelofs).

Alicia Boole Stott

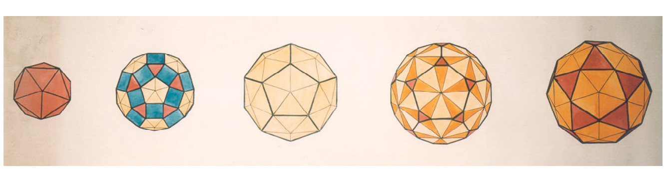
Alicia Boole Stott was the third daughter (of five) of the mathematician George Boole, who died when she was four years old. Unhappily placed with her grandmother and great-uncle, she rejoined her mother and sisters in London about seven years later, in poor lodgings. She had no formal education; yet despite this, Alicia's innate mathematical talent was awakened at about age eighteen, by C. Howard Hinton, a frequent visitor who married her older sister Mary. His interest in higher-dimensional space inspired her to study the subject on her own, and soon her knowledge surpassed his—she discovered the six regular polytopes in 4-space and constructed cardboard models of their sections. In 1890, she married Walter Stott, an actuary, who drew her attention to the work of P.H. Schoute at the University of Groningen; this led to a 20-year collaboration, publications of her work, and even an honorary degree. After Schoute died in 1913, she abandoned mathematics.

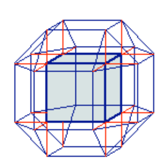


In 1930, Coxeter was at Trinity College, Cambridge studying for his Ph.D. under H.F. Baker; also, Alicia's nephew Geoffrey I. Taylor was a Fellow there. Realizing that his aunt and Coxeter had a common interest in Archimedean solids, he introduced them; quickly they became close friends and collaborators. Coxeter found her "Quite amazing. She had such a feeling for four-dimensional geometry. It was almost as if she could work in that world and see what was happening. She was always very excited when I had things to tell her ..., and she helped me in what I did." [12]

Saturday afternoons meant a "tea party" at Baker's house; these were in fact, research seminars with required attendance and presentations by his students. On one of these occasions, Coxeter invited his 70-year old "Aunt Alice", as he called her, to deliver a joint lecture. She brought along her models of sections of Gosset's 4-dimensional polytope (which Coxeter had rediscovered about that time) and donated them to the mathematics department (where they remain, at the Newton Institute of Mathematics). The University Museum of Groningen has her 100-year old models of cross-sections of the 600-cell (see above), her drawings of five of these (see below), and perpendicular sections of the 120-cell. Her models are colorfully painted and shellacked, with pins at vertices; her diagrams of their nets are in [27].

Coxeter's genuine affection for Alicia and admiration for her work is reflected in the two biographical essays he wrote in [4] and [9], and his frequent references in writings and lectures to her insight and her models. In chapter V in [1], he describes how to obtain the Archimedean solids from the Platonic solids in the traditional manner, using truncation. He then writes, "A far more elegant construction for the reflexible figures has been devised by Alicia Boole Stott [28]. Her method is free from any employment of distortion, and the final edge-length is the same as that of the regular solid from which we start. In the process called *expansion*, certain sets of elements (viz., edges or faces) are moved directly away from the centre, retaining their size and orientation, until the consequent interstices can be filled with new regular faces. The reverse process is called *contraction*. By expanding the cube, ...we derive the rhombicuboctahedron, 3.4^3 [below, right]. Mrs. Stott has represented these processes by a compact symbolism, and extended them to spaces of more than three dimensions, where they are extraordinarily fruitful."



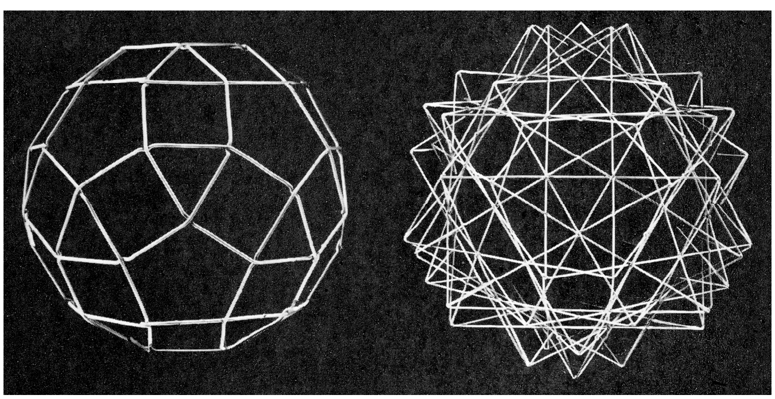


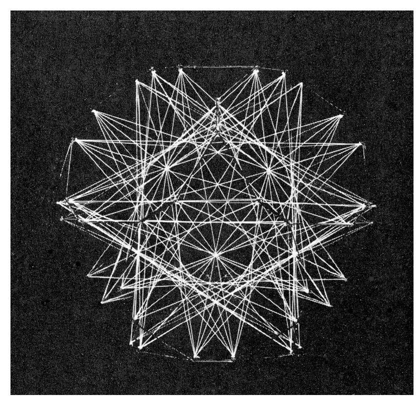
Michael Longuet-Higgins

Michael Longuet-Higgins (b. 1925), had the good fortune to have a mathematics teacher who introduced him, at age 11, to making cardboard geometric models of the Platonic solids. “So began a life-long love affair with polyhedra, and more generally, polytopes in space of *n* dimensions.” [15] In the summer of 1939, he and a friend made models of all the Archimedean solids. At about the same time, his brother Christopher and schoolmate Freeman Dyson, inspired by Coxeter's book *The Fifty-nine Icosahedra*, constructed models of all fifty-nine stellations of the icosahedron. This exercise led Christopher to discover several other compounds of polyhedra, and when Michael joined him at Winchester College (a British ‘public school’ for boys), they made cardboard models of all these.[1]

Fascinated by uniform polyhedra[2], the brothers set out to enumerate them, and found several not in the published literature. They could not know that earlier (from 1930-1932), Coxeter and his research student J.C.P Miller had also attacked this problem, and were reasonably sure their list was complete, but withheld publication since they lacked a proof. Between 1940 and 1942, Michael and his brother worked at the problem, and Michael devised a clever way to make models of galvanized iron wire that displayed only the principal edges of the faces of the polyhedra (the faces by definition are regular polygons or star polygons). No solder was used—where edges intersected, the wires simply passed over and under alternately and a small kink in one wire kept the other in place. At vertices, one crimped wire held the others meeting there; the springiness of the wire held the model together (see below, left).

From 1943 (while a mathematics student at Cambridge) until 1952 (now holding a Cambridge Ph.D. in geophysics), Michael produced wire models of all their discoveries, with the exception of the most complicated form, in which as many as 10 edges must pass through a single point, For this, he soldered a wire frame, painted it black, then used the frame to string white cotton threads to represent the edges (see below, right). There were 36 models in all, some representing several different uniform polyhedra, since faces can join edges in different ways.





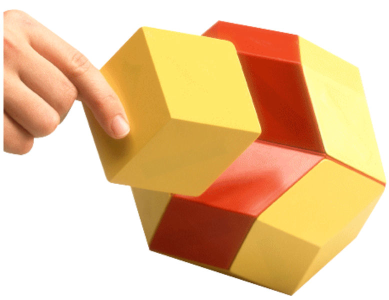
In 1952, while in Toronto for a physics conference, Freeman Dyson visited Coxeter and told him about the models; Coxeter and Miller were then in the final stages of writing up their monumental paper *Uniform Polytopes*, for which Miller was making drawings. Coxeter immediately wrote to Christopher, enclosing the list of uniform polyhedra he and Miller had compiled, and asked how many Michael and Christopher had discovered. Christopher confirmed they had found all but one on the list, namely the exceptional snub polyhedron, called by them “Miller's monster.” He also invited Coxeter and Miller to see Michael's models at the family home in Oxford, and those in Winchester. The visits were soon made, and Coxeter wrote to Michael (then studying in California), “Your parents gave us a splendid welcome

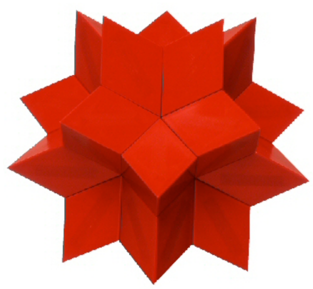
[1] All the models are still on display at Winchester.

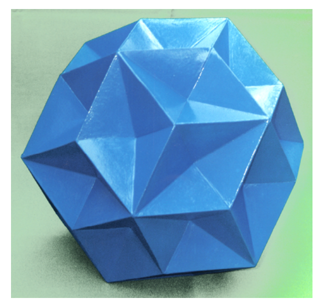
[2] A uniform polyhedron has all vertices congruent and all faces regular polygons or regular star-polygons.

and showed us the models... We admire them greatly. The last [previous page, right] was especially welcome, as Miller's drawing of it is not yet complete, and your white threads enabled him to detect an error before its consequences became serious." He then proposed that Michael become a third author of their treatise and that photos of his models of all the uniform polytopes appear there. Michael accepted, and became a full collaborator on the project, adding his own discoveries made from his models. After more than a year, all of Miller's drawings and Michael's models were complete; extensive correspondence during that time shows the care the authors took to make the best presentation, in both text and pictures. The Royal Society evidently also agreed that all the illustrations were important; none were omitted. [5]

For the next thirty years, as a full-time researcher in physical oceanography (1969-1989 a Royal Society Research Professor at Cambridge, and 1990-present at the Institute of Nonlinear Science, University of California at San Diego), Michael's model-making pursuits lay dormant. Then, in 1986, an article in *Scientific American* on quasicrystals awakened his interest. He began to experiment making models of "golden rhombohedra" that could be assembled face-to-face, in an attempt to fill space. These six-faced polyhedra are like stretched cubes, with congruent golden rhombs[3] as faces, and come in two varieties, the acute A_6 and obtuse O_6. His first models, of cardboard with velcro for sticking together faces, were unsuccessful (assemblages didn't pull apart easily!). Magnets glued to the inside of the cardboard were more successful. After making his own trial models of injected plastic with magnets embedded in the faces, he found a small firm of plastic molders near Cambridge that could produce these RHOMBO blocks professionally; he saw it as a potential educational toy[4].







In 1993, Michael sent Coxeter prototypes of the blocks, 10 of each type, enough to make a "Kepler ball" (a rhombic triacontahedron) (see above left); this intensified a continuing correspondence that lasted until Coxeter's death. Michael and Donald traded many insights on the triacontahedron, its stellations, and other geometric subjects throughout this period; the blocks and other models made by Michael provided impetus for new discoveries, including what he called the "Kepler star" (above, middle).[5] "Your RHOMBO blocks are very much more than a toy!," Coxeter wrote to Michael. When Michael sent Coxeter a cardboard model of a new polyhedron G_{60} discovered by Branko Grünbaum (above right), it so impressed Coxeter that he quickly changed the topic of an invited lecture in order to speak about it.[6]

At the Stockholm conference *Symmetry 2000*, Michael reports that Coxeter dramatically held up a Kepler ball made from the RHOMBO blocks, "then apparently fumbled and dropped it on the floor, where of course it flew into pieces. I was in the front row, and rushed forward to pick up the magnetic blocks and put them together again, so providing an impromptu demonstration. Did Donald drop the blocks on purpose? I believe he did, so as to give me a chance to demonstrate the toy. That would have been just like him."

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[3] The diagonals of a golden rhombus are in golden ratio, $\frac{1+\sqrt{5}}{2}$.

[4] RHOMBO is now available to the public; see www.rhombo.com

[5] See [10], [24] for more details on Coxeter's investigations on the triacontahedron. See also [13].

[6] A later wire edge-model of this polyhedron yielded even more insights; see [13].

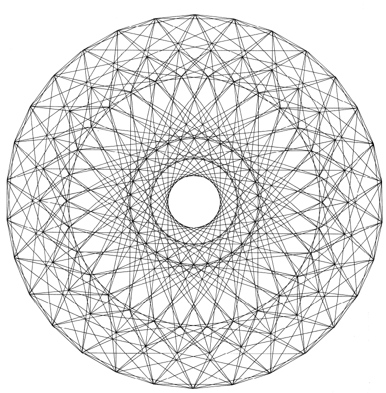
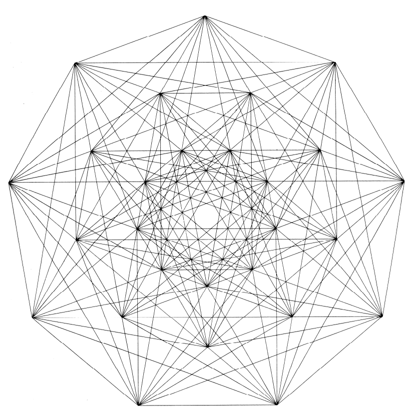
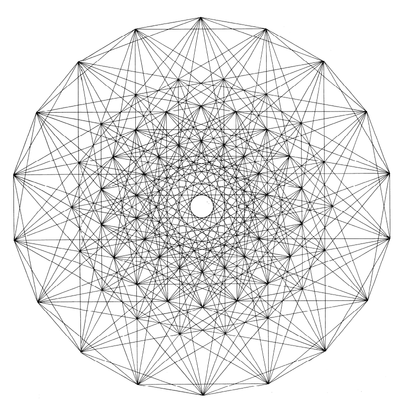
Peter McMullen

Peter McMullen became interested in regular polytopes in 1961 while a first-year student at Cambridge, not through a course, but because his father borrowed a copy of Coxeter's *Regular Polytopes* from his college library to keep Peter amused during Easter break. The frontispiece of the book was a beautiful drawing by S.L. van Oss of projections of the Petrie polygons of the four-dimensional polytope denoted {3, 3, 5} (the 600-cell). In chapter 13 on sections and projections, he found Coxeter's description of how to find the Petrie polygons of this polytope. Soon after that, McMullen sought out van Oss's papers in the library of the Cambridge Philosophical Society, and for several months, worked out from the projections of all the regular 4-polytopes the corresponding projections of their Petrie polygons, and drew them.

Each drawing took from one to three days to complete, beginning with a penciled version 12 inches in diameter, made with compass and protractor. Then with steel ruler and drawing pen, Peter carried out the precise (and tedious) process of inking each line, carefully lifting the ruler to avoid smearing, and waiting for the ink to dry before drawing the next line. During the next few years, he investigated and drew projections of other 4-polytopes, and did most of the research for his master's thesis and his published article on star-polytopes [17] before formally becoming a research student of G.C. Shephard.

In 1968, on a visit to Shephard, Coxeter saw McMullen's drawings and was deeply impressed. These, along with Peter's research, seem to have given Coxeter impetus to produce his book *Regular Complex Polytopes* (*RCP*) which was published in 1974. In order to accommodate the drawings in near-original size, Coxeter insisted that the book be an unconventional 10 in × 11 in. Not only were Peter's drawings an integral part of the book; he became a full collaborator in the exposition, with much of the last two chapters devoted to his work. Their friendship and mathematical interchanges continued over the years (McMullen is Professor of Mathematics at University College London); Coxeter requested a new drawing for the second edition of *RCP*, and also used several of Peter's drawings in other publications.

The large size and webs of fine lines in most figures in *RCP* prevented them from being reproduced here (photos do not do the drawings justice). Figure 4·8C (below left) is derived from the van Oss projection that got Peter started in 1961; this figure and Figure 12·4B (below, right) were drawn in 1968 at Coxeter's request. Figure 12·3B (below, center) is the 'Hessian polyhedron' in which the incidences of its diameters and planes of symmetry are the same as the incidences of the Hessian configuration of twelve lines and nine points in which points lie by threes on the lines, and four lines go through each point.

In the preface to *RCP*, Coxeter wrote "Most of these figures have recently become amenable to the technique of computer graphics, However, the skilful hand of Peter McMullen, using simply a steel ruler and a pen, can be seen in the[ir] delicate intricacy..." In the second edition, he calls McMullen's drawings "spectacular" and closes his preface, "In addition to their mathematical meaning, those pictures can be appreciated as abstract art."

A.G. (Tony) Bomford

Tony Bomford, born in India and educated in England, joined the Royal Corps of Engineers in 1944 and with their support, took further education and mathematical studies at Cambridge. He went on surveying and mapping expeditions in Tanzania, South Georgia, England, and Australia, where he eventually settled permanently. There he retired (early) in 1982 as senior surveyor in the Australian Division of National Mapping, in order to pursue fully his passion for adventurous travel, and at home, quiet avocations of hooking woolen rugs and cutting geometric models of wood. Tony was keenly interested in all things mathematical; he followed Martin Gardner's column Mathematical Games in *Scientific American* closely. In 1975 when Gardner reported a new tiling by pentagons, Tony made a hooked rug (#5) featuring that tiling and sent a photo to Gardner, who, knowing my interest in the pentagon problem, sent it on to me.

I played an indirect role in bringing together Coxeter and Bomford. In 1981, I included the picture of Tony's rug (with his permission) in my article in *The Mathematical Gardner*; consequently, he received the book, which also contained Coxeter's article "Angels and Devils." [8]. Here Tony, like Escher many years before, was struck by the hyperbolic tiling on which Escher's famous print was based. He wrote to me in December 1981, "Coxeter ... has joined the ranks of my heroes ... in particular Figure 7 of Escher's Circle Limit IV. Like Escher, I had difficulty in finding the centre and radii of more than the largest circles. ... I sent in turn for most of the references at the end of Coxeter's article and digested them and had the pleasure of following in Escher's footsteps in 'diligently pursuing these ideas' alternately drawing, then calculating, then doing a little algebra or trigonometry to produce a better formula for my calculations, then drawing again and iterating until finally I had a scaled drawing of a (6,4) tessellation about seven feet in diameter drawn and calculated to some fraction of a stitch at the outer radius." He closed his letter, "eternal thanks for leading me through Escher to Coxeter."[7]

Eleven months later, Tony sent me a picture of his beautiful rug #12, "hyperbolic spiderweb," its Turkey wool in hues of ivory, sand, orange, and brown, 2.18 m in diameter, and containing 64,242 knots. He also sent the picture to Coxeter, introducing himself and describing in detail his investigations inspired by the article, adding "There were periods when I thought you had been unkind to Escher, not answering his questions explicitly[8] but only sending him a few hints. I had advantages ... being by profession a surveyor and cartographer." He wrote that he "enjoyed and admired so much" *Regular Polytopes* and *Introduction to Geometry*. This was the beginning of a 20-year exchange of letters, full of personal news mixed with geometric observations, discoveries, recommendations, and questions. Coxeter's hand-written reply to Tony's first letter ended with a plea, "[A woman] has written to ask [me] some questions about (6,4) which I cannot readily answer. Will you please answer them and make her happy?" Tony gladly supplied her his list of 40 hand-calculated centers and radii for the fundamental triangle for (6,4); he also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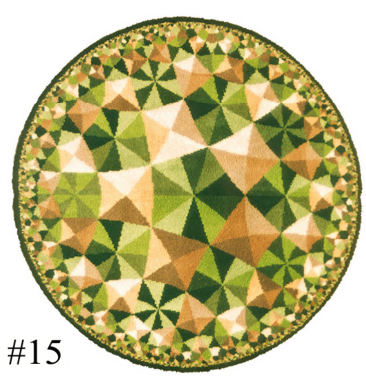
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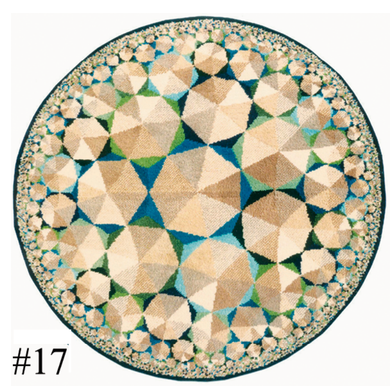
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[7] In my reply, I suggested that he contact Douglas Dunham to find out about computer generation of hyperbolic tilings (m, n).

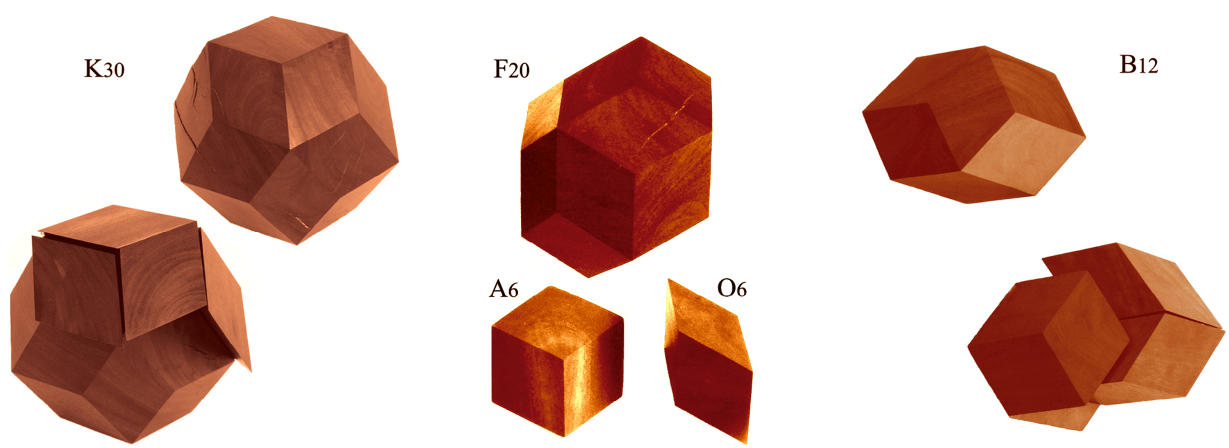
[8] See [26] for this correspondence between Coxeter and Escher.

sent the list to Coxeter, advising "If you get more enquiries from enthusiasts ... asking for a 'simple explanation how to construct the circles', [this] will enable you to send a quick reply, at the risk of spoiling the fun for the recipient." Bomford relished the challenge of working out problems himself, and finding sometimes unexpected results, which he shared with Coxeter. He was a master of trigonometry; once Coxeter had to apologize for an incorrect trigonometric calculation which Bomford couldn't verify.

In 1983, Tony got in touch with Douglas Dunham, who supplied him with his computer calculations of radii and centers of circles of (6,4), confirming all but two of Tony's calculations (which amounted to only 0.4 of a stitch). At Tony's request, he later sent calculations for the (7,3) and (5,4) as well as large-scale pen plots of the fundamental triangles. Soon Tony purchased a computer and proceeded to write his own programs to generate coordinates of radii and centers: first, program 'Dunham', then 'Hyperb', which used only Euclidean reflection, rotation, and inversion in an extended arc of the fundamental triangle. He "digested" Coxeter's article [6], using Figure 10 as a guide for coloring regions; his rugs were to have harmonious colors that revealed as many different tessellations as possible. From 1983-89, he produced roughly one rug per year, with all but one a hyperbolic tessellation. Rug #13, "hyperbolic lagoon", was another (6,4), #15 was based on the compound (4:5), and #16 and #17 were based on (3:7). Mathematical details on the rugs can be found in Dunham's article [11].

Throughout this time, Tony spent several months each year exploring remote parts of the globe, and for another couple of months of the year, occupied himself with cutting beautiful wood models of polyhedra. In his first letter to Coxeter, he wrote, "I would like to cut all the regular polyhedra ... and a series of sections of 4-polytopes and mount them on a window sill, so that as an engineer can get a 3-d picture from 2-d plans, my visitors might be able to get a glimpse into that 'new world of dazzling beauty' to which you refer." Coxeter supplied him with many references. Tony cut his first models in 1983 with a borrowed radial saw; soon he purchased his own saw, and immersed himself in learning, mostly through trial and error (and calculation), how to cut from red box, models of constant volume, so that they would be roughly the same size. "When [red box] is cut and polished ... it is so dense it sinks in water, and a well-polished hedron feels cool in the hand like a billiard ball." He shared his calculations, practical observations, frustrations (especially with cutting the "knife-edge" A_6), and discoveries, in lengthy letters to Coxeter, who had encouraged him to cut models of the golden rhombohedra and zonohedra that can be built from them, Belinski's B_{12}, Fedorov's F_{20}, and Kepler's K_{30}. He offered, and Coxeter accepted, a gift of these models, made with equal edge-length, so the small models could build up larger ones.[9]

In their last few years of exchanging letters, 1989-2001, Tony mostly told of exciting travel, and apologized for not finishing new rugs or doing much cutting of polyhedra; he shared a few mathematical curiosities with Donald. In fact, both of them were winding down; each died in the spring of 2003.



9 See the articles [24], [25] by Marjorie Senechal.

Acknowledgements

Photos of Alicia Boole Stott's drawing and models at the University of Groningen were provided by Irene Polo-Blanco; those of Tony Bomford's rugs and models were provided by his son Richard. I am grateful to them, the artists, their families, and friends who answered many questions and provided information and pictures. I also wish to thank Asia Ivíc Weiss, Philip Bomford, W.B. Raymond Lickorish, and Douglas Dunham for their help.

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