Donald Coxeter:
The Man who Saved Geometry

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Abstract

Siobhan Roberts met Donald Coxeter when she was a journalist at the *National Post* and writing a profile on the greatest living classical geometer. Shortly thereafter, she followed Coxeter to the last two geometry conferences he would attend — at Banff, Alberta, and then at Budapest, Hungary, in 2002 for a celebration of the 200th anniversary of the birth of János Bolyai (1802-1860), one of the discoverers of non-Euclidean geometry. Coxeter inspired many mathematicians over his long career. Roberts was similarly entranced by the man and his geometry, notwithstanding the fact that she was a geometrical innocent. Four years later, she is completing her biography of Coxeter. It is to be published by Penguin in Canada and Walker & Company (Bloomsbury) in the United States (as well as publishers in Italy, Japan, and Korea), in the fall of 2006. This piece is an excerpt from the introduction to the Coxeter biography.

Coxeter with one of his kaleidoscopes, circa 1960.

Whenever I dared mention I was writing a book about the man who saved geometry from near extinction — when the topic came up at a dinner party, for instance — the conversation usually skipped a beat or ten. Expressions around the room took on various shades of stupefaction as people entertained flashbacks of math class anxieties, fumbling with compasses and protractors and memorizing the Pythagorean theorem. Geometry was seared in their minds as a traumatizing experience, a subject enthusiastically abandoned, and as a result people branded themselves "anti-geometry," or at the very least, "geometraphobes." The conversation resumed only when one brave individual broke the silence with some variation on this rejoinder: "This guy saved geometry?!? WHY, on Earth, did he do that? He would have saved us all a lot of misery if he had let it die!"

I came across a more reassuring sentiment one weekend while visiting my sleepy hometown in eastern Ontario. I was driving down the main drag, a cultural promenade of fast food chains and car dealerships, when the billboard at the Di$count car lot caught my eye. It said, with a few letters slightly askew:

“WITHOUT GEOMETRY LIFE IS POINTLESS”

This is the rub with geometry — the study of points, and lines and shapes in space. We no more notice it than we notice the curve of the earth, yet if you take a closer look, geometry pops up where you least expect. It bids an invitation to a hypertext reality, a land illuminated with shapes and patterns, symmetries and reflections — geometry is everywhere and its impact is infinite. Geometric algorithms produce the computer-designed curves of a Mercedes-Benz, and films like *The Incredibles* by Pixar Animation. It’s in the molecules of our food and drugs — observed under a stereomicroscope, the spearmint molecule is the exact symmetric reflection of the caraway molecule; and Thalidomide, the pill designed to prevent morning sickness in pregnant women, proved so damaging only when it was ingested at bedtime and the sleeping hormones of the female body flipped the original structure of the drug into its mirror opposite, which was untested and thus caused unexpected birth defects.

My first experience with geometry in Grade Six didn’t make me either phobic or a precocious devotee. I was fond of my geometry kit (boys especially liked the compass because it could be contorted to look like a gun) and as a result I did well in math class that year. So well that I skipped Grade Seven math and went on to take all the math courses I could in high school. When university came I flipped the arts-or-sciences coin for my field of study and enrolled for an obsolete BA in history. I gave mathematics nary another thought, until I came across the man who saved geometry, one Donald Coxeter.

When I first met Coxeter, there were a number of qualities about him that quickly and thoroughly drew me in. It was equal parts the man, age 95, at once an absent-minded (at that age, anyway) and mastermind professor, still operating under his own steam, though fortified by a bedtime elixir of Kahlua coffee liqueur, peach schnapps, and soy milk; and it was Coxeter the mathematician, muse to M.C. Escher and Buckminster Fuller. Escher had long been looking for a way to convincingly portray infinitely, and his mental block was broken when he set eyes upon one of Coxeter’s geometric illustrations. Thereafter he credited as the inspiration for his Circle

Limit III drawings, and when he was working on them Escher was known to say, "I'm Coxetering today."

And Buckminster Fuller — Bucky as Coxeter called him —dedicated his book *Synergetics, Explorations in the Geometry of Thinking* [1] to Coxeter with this high praise:

By virtue of his extraordinary life's work in mathematics,
Dr. Coxeter is the geometer of our bestirring
twentieth century, the spontaneously acclaimed
terrestrial curator of the historical
inventory of pattern analysis.
I dedicate this work with particular esteem for him
and in thanks to all the geometers of all time
whose importance to humanity
he epitomizes.

Coxeter considered this dedication a bit of name-dropping, and suggested that Fuller would have done better to consult a mathematician in the writing of the book. Nonetheless, Coxeter was a fan of Bucky's geodesic domes, and when Coxeter's cottage burnt to the ground in a lightning storm, he contemplated building a geodesic dome cottage in its place.

Coxeter's precocious childhood was also quixotic and alluring. He won a scholarship to Trinity College, Cambridge, only after his tutor forbade him from thinking in the fourth dimension while he focused on the foundations of mathematics that were on the entrance exam. But this mathematical success came after some significant dabbling with music — both playing and composing. Both Coxeter's parents were artists, his mother a painter, and his father a sculptor and a baritone singer. Coxeter learned to play the piano before the age of 10 when his father's musician friends came by the house to practice. But by this age he was also composing scores and lyrics. An operatic piece he called "Fairy Song" went like this:

There are fairies at the bottom of our garden,
They often have a dance on summer nights.
The butterflies and bees make a lovely little breeze,
And the rabbits stand about and hold the lights.

Did you know that they could sit upon the moonbeams
And pick a little star to make a fan
And dance away up there in the middle of the air?
Well, they can.

Donald penned arrangements titled "Autumn" and "Devil" as part of a larger opera, "Magic," composed as incidental music for G.K. Chesterton's play of the same name, even going to the trouble of indicating where in the play his music should rise in and fade out. Most of his pieces were composed as Christmas or birthday gifts, and dedicated to his mother or father. Coxeter discussed the link between his early mathematical and musical inclinations in a two-part radio documentary, aired in 1997 on a Canadian Broadcast Corporation radio program about math and art. "The story goes that I, at an early age, liked to look at the numbers that were published in the financial news because I was fond of numbers, as such. But I suppose it was connected with music. When I met Ernest Galloway, who was a violinist and composer, a great friend of my father, he taught me the theory of music, intervals, harmonics and so on, and showed me how to write music," Coxeter said, adding that these music lessons probably took place when he was six or seven years old. "I was interested in the structure of the notes of music," he continued. "That was somewhat mathematical. I think it's clear that one has to regard [music] as being mathematical: the 12 semitones in the octave and the 8 diatonic notes and how they are different…The pleasure I got from writing music went over rather naturally into mathematics. I got the same kind of euphoria from a successful piece of mathematical rediscovery than I formerly did in writing a piece of music." Donald's mother pursued an evaluation of her son's musical talent. She took him to see Gustav Holst. "I don't know how she got to him," Coxeter recalled, "but she took me along and I showed him some of the music I had written, and I played a little bit on the piano. On the whole he thought it was rather poor." It should be noted that during a recent celebration of Coxeter's life, his compositions were dug out, evaluated by professional musicians for a performance, and about 20 per cent were more than decent enough to be played, and in fact were quite complex and challenging arrangements. But Coxeter and his mother received much the same evaluation from a visit to another composer C.V. Stanford. His reply was, "Educate him first."

Luckily, then, Coxeter had geometry as a backup. Coxeter described geometry with his ditty of a definition: "Geometry is the study of figures and figures" — figures as in shapes (circles, triangles, dodecahedrons), and figures as in numbers. Coxeter's geometry was the classical style, a style in which hands-on and visual reasoning — using antiquated treasures like triangles, circles and polyhedrons — were the crucial tools for exploring and finding answers. And at the very heart of Coxeter's appreciation of shapes is the notion of symmetry. Coxeter's geometry is all about searching for symmetries of a shape. Prefacing one of his most popular books, Introduction *to Geometry* [2], Coxeter states: "The unifying thread that runs through the whole work" — and indeed his whole life and career — "is…in a single word, *symmetry.*"

"All of mathematics is the study of symmetry, or how to change a thing without really changing it," said Coxeter in another CBC radio program, this one in 1972 on symmetry, and called "Mathemagic, the Ambidextrous Universe." "It is symmetry, then, in its various forms, which underlies the orderliness, laws, and rationality of the universe, and thereby also the language of mathematics." We usually say something is symmetrical if it looks like its reflection in a mirror, or more generally like its image under some other geometrical operation that preserves measurement. Etymologically the word deconstructs to "sym," meaning together, and "metry," meaning measure, and implies that different parts "measure together." Symmetrical people's left feet are as long as their right.

Symmetry is ubiquitous. In a generalized sense, the music of Bach is symmetrical, as is the art of Leonardo da Vinci, the metrical rhythm and rhymes of poetry, and the turrets and arches of architecture. Symmetry, in more precise meanings, suffuses all of science — biology, chemistry, physics, and cosmology. In physics — the study of the physical systems of nature — symmetry exists if a change can be made to a system but yet the system remains the same before and after the change. The change made to the system is called a symmetry operation or transformation. If the system stays the same when subjected to a symmetry operation, the system is invariant or symmetrical. The sphere, for example, can be rotated in an infinite number of ways and it always remains exactly the same; a sphere is invariant under an infinite number of symmetry operations. Astrophysicist Fritz Zwicky (1898-1974) was notorious for calling people "spherical bastards" if he found them uninteresting and dislikeable — no matter which way he considered these people they were equally offensive. Infinite symmetries, then, themselves are not so interesting. They are predictable and thus hold less appeal than the discrete symmetries that Coxeter liked investigating. Coxeter used kaleidoscopes to produce figures like the Platonic solids, and generate their symmetries through mirrored reflections. He extended this method of investigating shapes into multiple dimensions, where shapes rotate and reflect, replicating their properties and appearance in the hall of mirrors that is hyperspace.

A nice initiation to Coxeter's view of the world was during one of our many "research" outings, whereby I followed Coxeter around in an effort to immerse in his shapely reality. We attended a reception for new inductees to the Royal Society of Canada, a club of distinguished scientists and scholars modeled after the Royal Society of London (Coxeter was a fellow of both, in the latter in good company with alumni Albert Einstein and Sir Isaac Newton). The gathering was held at one of the University of Toronto's well-appointed mansions in the city's riche

Rosedale neighbourhood, just up the hill from Coxeter's very cubic house that was perched defiantly on a ravine. Waiting for the formalities to begin on this cold, dark winter evening, Coxeter and I sat in the library, sipping wine, warming by the fire. Between the approach of fans and well-wishers, Coxeter gestured into the middle distance and asked, "What shape is that table?" I knew it was a trick question. But I said what I saw. It was a circle. He corrected me: If I was suspended from the ceiling looking down upon the table, then it would be a circle. But from our coordinates across the room his perspective was slanted and transformed. He saw the table as an ellipse, adding as a footnote that he had written a paper on this subject, titling it poetically, "Whence does a circle look like an ellipse?"

This was classic Coxeter: ruminating about the romance of shapes, ellipses and circles, hexagons and icosahedrons. He delighted in the geometry of froth, sponges, honeycombs and sunflowers. During his professorial days, Coxeter picked towering sunflowers from his garden, stalks as tall as the man himself, and toted them along on the city bus to the university, where he employed them as teaching devices. One student recalled that Coxeter dabbed a dot of glossy red nail polish on each of the sunflower's seeds, accentuating the geometrically perfect golden ratio of the seeds' graceful whorl. His passion for shapes was motivated exclusively, almost with an elitist bent, by beauty. I once asked him why he kept at it, why he was still working even long into retirement when he was paid only his pension? "No one asks artists why they do what they do," was his retort. "I'm like any artist. It's just that the obsession that fills my mind is shapes and patterns."

It might seem with his rarified passion for shapes that the demise of Coxeter's classical geometry, like the loss of, say, Latin, would pass virtually unnoticed from this world. Surely, in an age of supercomputers and superstring theory, the crude tool of classical geometry has become obsolete. Hanging around with Coxeter, I soon knew better.

As I continued to follow Coxeter around, however, to mathematics conferences in Banff and Budapest, it became clear that the modern raison d'être for geometry was not merely as a paean to the beauty of shapes.

A geometer by the name of Walter Whiteley was one of the first clues. In contrast to those geometraphobes at the dinner party, Whiteley, director of applied mathematics at York University in Toronto, devotes a considerable amount of time and energy to pleading geometry's case. He wrote a paper addressing "The Decline and Rise of Geometry in 20th Century North America," which he delivers as lengthy Power Point presentation full of visuals. "Do these tracks of a bicycle indicate it was traveling forward or backward?" he asks. "Do you see this face as smiling or frowning?" It all seems more Rorschach than geometry. But Whitely calls it "learning to see like a geometer." And Whiteley warns that if geometry had met its demise, the consequences would not be small. He reckons a "geometry gap" would haunt Western civilization for generations to come (for reasons we won't get into, geometry was safer some parts of Europe, such as Russia and Hungary). Without classical geometry, as without Mozart's symphonies or Shakespeare's plays, our culture, our understanding of the universe, would be impoverished and incomplete.

Having delineated these parameters on the pure and applied importance of geometry in general, I next had to decipher what was amassing in my mind as the mythic proportions of Coxeter's story. It was this awe-inspiring fact that he was introduced not only as "the world's greatest living classical geometer" but also, on occasion, as "the man who saved geometry." This savior designation seemed like a bit of hyperbole, warranting the equivalent of a mathematical proof. My first task was to update my knowledge of geometry. I enrolled to audit Whiteley's course "Introduction to Geometries." The textbook stated that "a formal proof as we normally conceive of it is not the goal of mathematics — it is a tool — a means to an end." A proof, it said, was "a convincing communication that answers — Why?" Indeed, this was the query I often faced. Why, and how, did Coxeter save classical geometry?

It all had to do with context and timing. Right around the time Coxeter chose classical geometry for his career, circa 1930, the visual tradition of this genre, appreciating circles, triangles, and polyhedra, was exiting its golden age (the peak not being Euclid's day, but the nineteenth century). But as E.T. Bell [3], historian of all things mathematical, pronounced in 1940: "The geometers of the 20th century have long since piously removed all these treasures to the museum of geometry where the dust of history quickly dimmed their luster." Coxeter's geometry was in decline. Geometry was being recast, like a disappointing remake of a cinematic classic, in an abstract format. The "modern" discipline of geometry was being subsumed by algebra and analysis. It was all equations, no shapes.

Coxeter, meanwhile, must have been wearing blinders. He persevered with the shapes he loved. He had a particular passion for polytopes, so much so that during his stint at Princeton in the early 1930s he became known as Mr. Polytope. Polytopes are a family of shapes extended into the multiple dimensions of hyperspace. They include the sub-species polygons, figures on the two-dimensional plane having three or more straight sides and angles. Coxeter pointed out in one of his books that, "Everyone is acquainted with some of the regular polygons: the equilateral triangle which Euclid constructs in his first proposition, the square which confronts us all over the civilized world, the pentagon which can be obtained by making a simple knot in a strip of paper and pressing it carefully flat …and used as the base for the Pentagon Building near Washington… the hexagon of the snowflake, and so on." The family of polytopes also includes the sub-species of polyhedrons, solid figures of the three-dimensional plane. The most famous polyhedra, perhaps, are the five Platonic Solids — the tetrahedron, cube, octahedron, icosahedron, and dodecahedron. A polytope is an extension of either a 2D polygon or a 3D polyhedron into any, or "n"-dimensions.

The creatures that result are psychedelic renderings of a Rubik's Cube on acid. And these shapes really can be called creatures. Coxeter's house was a veritable zoo of them, the entire place overtaken by a gaggle of muliticoloured spiky models. He had polyhedron art on his wall, he had polyhedron lamps and polyhedron bookends, polyhedrons were behind glass, filling proper display cases, and they encroached on bookshelves, the fireplace mantel, side tables, and sometimes the dining room table. His books *Regular Polytopes* [4] and *Regular Complex Polytopes* [5], to name only two, were mathematical classics, bestsellers that seem to be forever in print. Coxeter and his books could be called the geometrical analog of Darwin and his *Origin of Species*. Because Coxeter did for polytopes what Darwin did for organic beings — he classified and enumerated all the symmetries of polytopes that could be found to exist. His findings are invaluable mathematical tools, known as Coxeter numbers and Coxeter groups, tools that are considered by some mathematicians to be as essential as numbers themselves. Indeed, Coxeter groups provided an invaluable bridge linking geometry to algebra, and in doing so broadened the scope of each. In particular, however, this served to make geometry relevant once again.

Working away, traveling and publishing, Coxeter slowly but surely developed a hearty band of followers. On a snowy January evening in 1959, he took the night train from Toronto to Philadelphia to give a talk, refining his paper as he travelled. The following day, he noted in his diary: "About 40 broke into spontaneous applause after my 10 minute lecture on 'Close Packing and Froth.'" The next month Coxeter gave a version of the same lecture to 70 schoolteachers. Two months later again he spoke to a group of gifted school children on "Close Packing of Spheres." This time he drew upon a seventeenth-century paper with a title he figured his young pupils would find amusing. It was called, "Vegetable Statics." It pondered how many peas, if you squished a lot of peas into a large cubic pod, would abut against a central pea.

In that same year — the year that Donald Coxeter propagated to all who would listen on the subjects of optimal packing spheres, as well as the beauteous properties of triangles and polyhedrons and Fibonacci numbers (this time using a pineapple as a prop) — something quite to the contrary was taking place across the Atlantic.

A conference convened in Royaumont, France, on the urgent need for reform in the nation's mathematics education. An infamous French mathematician rose from his seat during one session, pounded his fist on the table, and screamed:

“A bas Euclide! Mort aux triangles!”

Translation: “Down with Euclid! Death to Triangles!”

According to mathematical legend — as I learned it — this war cry came from the heart of Nicolas Bourbaki. It was representative of Bourbaki’s brawn, if not so much his brain. Bourbaki was an enfant terrible of a modern mathematician who thought mathematical education in France was falling behind the international standard. He wanted to revolutionize how mathematics was taught. In so doing he sought to flatten shapes, to stamp out the use of triangles and circles. Bourbaki set out to write an algebraic encyclopedia of mathematics without diagrams. The Bourbaki aversion to shapes and diagrams was defended as serving the interest of purity. All mathematical results had to be reached by intellectual reason alone — by rationality — rather than by the senses. Our visual perception of the world is unreliable, he argued, our eye leaves us victim to subjectivity and error.

A *Scientific American* [6] article not two years before the Royaumont conference reported that Nicolas Bourbaki and his revolutionary approach, despite its supposed parochial agenda, had taken not only France but also the international mathematical community by storm. “The legends about him are many, and they are growing every day,” it observed. “Almost every mathematician knows a few stories about him and is likely to have made up a couple more. His works are read and extensively quoted all over the world. There are young men in Rio de Janeiro almost all of whose mathematical education was obtained from his works, and there are famous mathematicians in Berkeley and in Göttingen who think that his influence is pernicious. He has emotional partisans and vociferous detractors wherever groups of mathematicians congregate.”

“Ah, yes,” recalled Coxeter when I mentioned Bourbaki and the “Death to Triangles!” incident. He was cool and calm, with the retrospection of old age. “Everyone is entitled to their opinion,” he said. “But Bourbaki was terribly mistaken.”

As the *Scientific American* article concluded in a climatic final sentence to the introduction: “The strangest thing about [Bourbaki], however, is that he does not exist.” In reality, Nicolas Bourbaki, the mathematical international community learned, years after his first appeared in print, was a pseudonym for a secret society of mathematics, the crème de la crème of French mathematicians. The society was founded in 1935 and continued through to the late 1970s before the movement lost steam. In its heyday, the Bourbaki approach was very popular. Benoit Mandelbrot, the inventor of fractals, experienced the Bourbaki method first-hand. Now a professor emeritus of mathematical sciences at Yale and a fellow emeritus at International Business Machines Corporation, Mandelbrot embarked on his university education in France just as Bourbaki imposed his influence. And Mandelbrot’s uncle, Szolem Mandelbrojt, was at the time a devoted Bourbaki follower. He encouraged his nephew to follow this new and austere mathematical method. But also thanks to his uncle, Mandelbrot knew that Bourbaki had a strong bias against geometry. The young Mandelbrot was disheartened. The visual component of geometry was his inspiration. “I called my self a geometer,” he recalled. “For [my uncle], geometry was essentially dead except in mathematics for children, and one had to outgrow it to make a genuine scholarly contribution.” So Mandelbrot drifted out of mathematics. He dabbled in economics, engineering, physics, and physiology, intermittently being lured back into geometry. In the late 1940s Mandelbrot was at Caltech on a two-year scholarship, when who turned up to give a lecture? Dr. Coxeter. “He had a very strong style, such a love for the complexity of geometric shapes,” recalled Mandelbrot of Coxeter’s talk. “I found Coxeter a great reassurance.”

Marjorie Senechal, a professor in mathematics and the history of science and technology at Smith College in Northampton, Massachusetts, puts Coxeter’s role countering Bourbaki in blunter terms: “Coxeter saved me from Bourbaki,” Senechal says. “For me, Coxeter was the antithesis of Bourbaki. He kept at geometry when it was unfashionable. During the long, lean years — the dark ages — Donald kept the flame alive and encouraged us, kept us going." She even goes so far as to hypothesize that Coxeter was isolated because he engaged in a field that was scorned, denigrated as "playing with toys," and considered second-rate math. Certainly, no one in their right-angled mind would ever stake a career on it.

Yet Coxeter did, and now the pendulum of his classical geometry is swinging back. Geometry's visual tradition is proving relevant in almost every niche of science. Says Walter Whiteley, "If you do a search on the Internet with 'symmetry and viruses,' the images of viruses that come up look exactly like something of Coxeter's or Escher's. You look at it and say, 'Gee, that's just a beautiful piece of geometry.' But they are colour-coded proteins of a virus. There's a very common one in the shape of the icosahedra which you often see in Donald's books — it's the common cold."

Another example provides evidence of Whiteley's "geometry gap." Sir Harry Kroto, who, together with Robert Curl and Richard Smalley, won the Nobel Prize for their discovery of Carbon 60, confirms that had they been familiar with Coxeter's work, his team's long hunt for the shape of Carbon 60 would have been much shorter. Previously, there were only six forms of extant crystalline carbon, including graphite, as used in pencil lead, and diamond, but scientists had long speculated there was another. The structure finally discovered by Kroto et al looks roughly like a soccer ball, or one of Buckminster Fuller's geodesic domes, with 20 hexagonal surfaces and 12 pentagonal surfaces, each a carbon atom. C60, nicknamed the Buckminsterfullerene, is currently being researched as the superconductor of the future with potential applications as a delivery mechanism curing cancer and AIDS. It is one of the regular complex polyhedra that Coxeter was so fond of working with, the truncated icosahedron. Since the discovery of C60, a bit of research on Kroto's part into extant knowledge quickly brought Coxeter's work to light as the definitive reference. As a result, Kroto says the footprints of Coxeter's work are all over his subsequent C60 research on giant Fullerene molecules. He set to work trying to construct C240, C540, and C6000, with a copy of Coxeter's *Regular Polytopes* as their manual.

Coxeter's work also played a role in the invention of the modem that allows us to communicate so effectively. And László Lovász, a Hungarian who is now mathematician-in-residence at One Microsoft Way in Redmond, Washington, points out that Coxeter's interest in mutually tangent circles (on which Coxeter delivered his last paper, at the Budapest conference) is part of a hot topic, related to the kind of data-mining technology that runs e-commerce engines such as Amazon.com, and the American government's surveillance software such as MATRIX, or Multistate Anti-TeRrorism Information eXchange. These applications of Coxeter's work are, albeit, indirect and inadvertent. Coxeter played a grandfathering role. Working away through those dark ages of geometry, he created a repository of invaluable insights about shapes and how they behave in higher dimensions that serve as a point of reference, a database of sorts, for future scientists to consult.

Over the course of his career, Coxeter ran hot and cold when it came to applications of geometry. He expressed an interest in geometry's unexpected appearances in nature, and conversed with chemists or biologists. But beyond natural applications, he was nonplussed. Later in life, certainly, he was blissfully oblivious to these broader reaches of his work. As far as the modem was concerned, he deplored computers, never touched one and had his son-in-law send his emails. When he found out about modern applications he seemed to regard them with apathy or disdain, like a letter from a long-lost relative with whom he had little interest in getting in touch.

In coming to grips with the omnipotence of geometry, Coxeter's in particular, I seized upon a statement made by Brian Greene, a superstring physicist at Columbia University in New York, and author of the Pulitzer-nominated book, *The Elegant Universe* [7]. "There is perhaps no better way to prepare for the scientific breakthroughs of tomorrow," said Greene, "than to learn the language of geometry." He was referring to the conundrum of modern physics: physicists' today have picked up where Einstein left off in the search for a Grand Unified Theory, a single theory that explains everything. They want a theory that covers everything from the shape of our galaxy and all the alien galaxies (surely, they have to be out there) to the shape of the smallest nano-sized spec of nothing. This theory seeks to unite Einstein's general theory of relativity, explaining the large-scale properties of the universe, with quantum theory, explaining the matter and energy on an atomic and subatomic level. The trouble with these two laws of existence that we currently hold true is that they are mutually exclusive. They cannot both be correct. But we may yet escape the destiny of being reduced to, as Einstein put it, a cosmological crapshoot.

The latest epiphany, over the last quarter century, is superstring theory. And just to make the unfathomable string theory more complicated, it turns out there are actually five variations on string theory— each supposing in its own nuanced way that tiny strings comprise new, and smaller than miniscule dimensions of space. These five versions, however, have been brought together to form an even grander synthesis plot, known as M-theory. Edward Witten, the big-wig mathematical physicist behind M-theory, based at Princeton's Institute for Advanced Study, has his own ditty of a definition for his field of study — the "M" stands for matrix, magic, mystery, and another m-word he recently added to the list: murky. That is to say that superstrings and M-theory hold promise, but they are still just theoretical children. They are still a puzzle. It is interesting to note, though, that Witten expects that the physics of string theory and all its infinitesimally small dimensions will ultimately evolve into new branch of geometry.

String theory came up once when I was interviewing Coxeter. We were talking about *The Adventures of Alice in Wonderland* [8], one of his favourite pieces of literature. He especially liked the "Jabberwocky" passage. The way he said that word— "Jabberr-wOckAy!"— was with such relish, and he could recite entire stanzas with the same dramatic intonation, cranking the volume on his otherwise sedate self. Coxeter often dipped into his ratty copy of *Alice.* And as much he liked reading it aloud himself, he also recruited others as the readers because sometimes he preferred to sit back in his suitably majestic wing-backed chair and simply listen. I asked Coxeter, one day after I read him some Jabberwocky, how hadn't tired of it by now, why he reveled in it so.

"It's like reading about a part of mathematics that you know is beautiful," he explained, "but that you don't quite understand. Like string theory. That's as much a mystery to me as it is anyone else who can't make head nor tails of the eleventh dimension."

Therein, unwittingly, I think Coxeter was onto something. The enduring problem with string theory is that string theorists themselves can't even explain it. They shrug their shoulders and say, 'It might be right, or it might be wrong.' The trouble is the dearth of hard evidence since it occupies, as Coxeter alluded, a mere eleven dimensions, which are too microscopic to see. But the hypothesis is that within these microscopic eleven dimensions resides a new species of subatomic particles, known as supersymmetric particles, or "sparticles." Physicists are trying to detect traces of these sparticles at places like Fermilab in Batavia, Illinois (45 miles west of Chicago), and the CERN in Geneva — home of the world's largest particle accelerator and essentially the centre of the universe for determining the content of the universe when it was a trillionth of a second old (despite this status, they are building a new particle accelerator, the Large Hadron Collider, which, when it is switched on in 2007, will probe even deeper into matter and smash nuclei together with even more collision energy). The long hunt for supersymmetry, like the hunt for Carbon 60, is on.

This made me wonder: it sounds far-fetched, but could Coxeter's templates of the symmetries of shapes in multiple dimensions possibly unlock part of puzzle of the unified theory of everything? I entered "Coxeter and M-theory" into Google. The first thing that popped up was an academic paper by Marc Henneaux, a specialist in black holes at the Free University, Brussels, and director of the Service de Physique Théorique et Mathématique. His paper was titled, in big bold letters,

"PLATONIC SOLIDS AND EINSTEIN THEORY OF GRAVITY: UNEXPECTED CONNECTIONS"

As it turned out, it wasn't so much an academic paper as it was blurbs from another Power Point presentation. It read, in part:

GRAVITATION = GEOMETRY
Einstein *revolution: gravity is spacetime geometry*
General relativity has proved to be remarkably successful …but there are …

PROBLEMS

General relativity + Quantum Mechanics = Inconsistencies
(e.g., infinite probabilities!)

Synthesis of both should shed light on the first moments of universe (« big bang »), on black holes, and on the problem of why the vacuum energy is so small.
Towards a solution: string (M-)theory?

SYMMETRIES: THE KEY?

Symmetry = invariance of the laws of physics under certain changes in the point of view

What are the underlying symmetries of M-theory?

Platonic solids*: the golden gate to symmetry*

Platonic solids, of course, are the building blocks of geometry and the toys Coxeter played around with so often in his work. And sure enough, a little further into the notes, Coxeter's work was cited:

Coxeter groups may thus signal a much bigger symmetry.

"Coxeter's work does make an unexpected appearance in Einstein's theory of gravity," confirmed Henneaux, when I called him up in Brussels to inquire. When I hung up, I thought to myself, 'This would seem to answer the question of why the loss of Coxeter's classical geometry would be immeasurable, existentially infinite, on a universal scale to the order of a dimension as yet unknowable.' Ipso facto, then, this Coxeter story, I figured, was worth looking into.

References

[1] Buckminster Fuller. *Synergetics*, MacMillan Publishing Co. Inc., New York, 1975.
[2] H.S.M Coxeter. *Introduction to Geometry*, John Wiley & Sons Inc., New York, 1961.
[3] E.T.Bell. *Men of Mathematics*, Victor Collancz Ltd, London, 1937.
[4] H.S.M Coxeter. *Regular Polytopes*, Corrected 2nd ed, Dover Books, New York, 1973.
[5] H.S.M Coxeter. *Regular Complex Polytopes*, Cambridge University Press, 1974.
[6] Paul R. Halmos. "Nicolas Bourbaki," *Scientific American*, pp. 88-99, May 1957.
[7] Brian Greene. *The Elegant Universe*, Vintage, New York, 1999.
[8] Lewis Carroll. *The Annotated Alice, With an Introduction and Notes by Martin Gardner*, Clarkson N. Potter, Inc., New York. 1960.