A Geometric Analysis of the *Seven Heavens*

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Abstract

The *artesonado* ceiling in the throne-room of the Alhambra is known as the *Seven Heavens*, since it appears to represent the path traveled by the soul, passing through the seven heavens of the cosmos until reaching at the peak the eighth heaven of the Muslim paradise. This paper will discuss and analyze the geometric structure of the ceiling and its eight different types of star polygon designs. Using only the geometer's tools of straightedge and compass (or the electronic equivalent, the *Geometer's Sketchpad* software), skeletal versions of the star polygon designs will be reconstructed and classified according to the symmetry elements permitted.

Introduction

The Alhambra, the most celebrated tourist attraction in Spain, is also one of the most famous examples of Islamic art and medieval Islamic culture on the Iberian Peninsula. A fortress enclosed by a wall, it contains palaces, houses, streets, mosques, baths, gates and towers. Its name, derived from the Arabic phrase, *Qal'aa al Hamraa*, meaning "Red Fort," is attributed to the color of the fortress walls, constructed of the red clay and stone from the surrounding area.

From 1238 to 1492, the Alhambra was the seat of government for the Nasrid dynasty, the last sultanate on the Iberian Peninsula. Here, the Comares Palace, completed by Yusuf I (1333 – 1354) during the 14th century, was built on what remained of the first Nasrid palace, built by Ismail (1314 – 1325). During the second reign of Muhammad V (1362 – 1391), considered the Golden age of Nasrid rule, many buildings were erected and decorated with geometric mosaic tiles, vegetal patterns, inscriptions and stucco, giving the Alhambra its present appearance.

The daily life of the court at the Alhambra, took place in the royal quarters of the Comares Palace complex, where the sultan and members of his close family lived. The Comares Tower – the largest and highest external tower of the Alhambra with a height of 45 meters – houses the throne-room (also known as the Hall of the Ambassadors). The room is 1.3 meters square and 18.2 meters high. In the three outer walls of the room (other than the wall containing the entrance), there are nine alcoves with windows overlooking the valley below. Eight of alcoves are the same size, and are covered with four different geometric designs (two of each). The ninth recess, in the center of the northern wall directly opposite the entrance, is slightly larger and more lavishly decorated than the other eight. In this ninth alcove, at eye level, just above the lower part of the dado, an iconographic inscription may be found in the form of a poem. Especially composed for this room in the building by Ibn Zamrak (the last, great poet of Muslim Spain), it states "…for my lord, the favorite [of God], Yusuf, has … chosen me as the throne of his rule…" [1]. This ninth alcove is believed to be the position of the sultan's throne, thus giving the room its name.

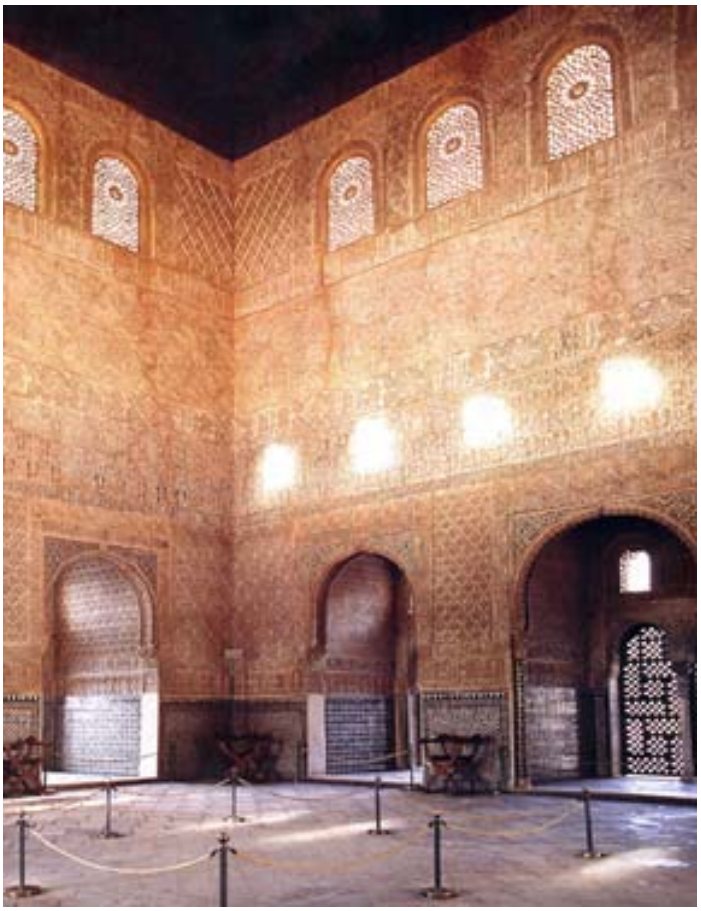


Figure 1: *View of the throne-room, with the ninth alcove to the far right*

The walls of the throne-room are completely covered with geometric mosaic tiles, vegetal patterns, and inscriptions in rounded (*naskhi*) and straight (*kufi*) calligraphy. Although faded now, these walls were once brightly colored, according to a wooden panel recently found at the Alhambra which describes the colors and shapes of the geometric designs. Above the stucco, along the four sides of the ceiling and starting on the north side, runs the entire *Sura* LXVII, the *Sura* of the Kingdom from the *Qur'an*. Of particular note are the last five verses of the *Sura*:

> "Blessed is He in whose hand is the Kingdom, He is powerful over everything, Who created death and life, that He might try you which of you is fairest in works; and He is the All-Mighty, the All-Forgiving, Who created seven heavens one upon another. Thou seest not in the creation of the All-Merciful any imperfection. …" [1]

This S*ura* inscription is embodied in the throne-room's wooden ceiling known as the *Seven Heavens*. Directly above the inscription, the ceiling takes the shape of a stepped pyramid, with each of the four sides composed of three wooden panels at varying angles to one another. These trapezoid-shaped wooden panels are nailed to timber roofing, which form the ribs of the vault. The vault is then supported from the walls by beams. The conception and execution of this ceiling is highly unusual and is unique to this room at the Alhambra; no other existing ceilings in Islamic Spain were constructed in this manner. On the next page, cross-sectional views of the pyramidal-stepped ceiling of the throne-room with the cupola in the center are provided in Figures 2. and 3. These figures show smaller sections of an elevation plan taken from the book, *Impressions of Granada and the Alhambra* [2].

Figure 2: *Sketch of a cross-sectional view of throne-room and the Seven Heavens ceiling from the book, Impressions of Granada and the Alhambra*

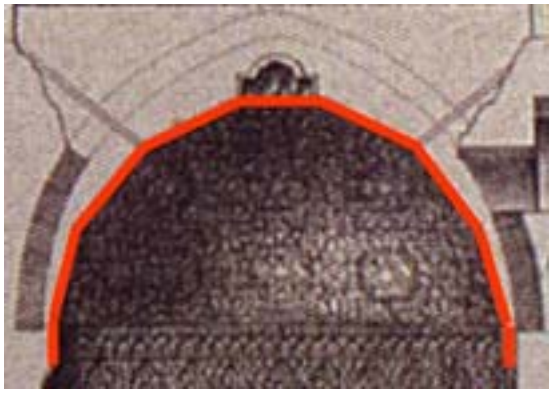


Figure 3: *Accentuated view of the angular planes and the cupola of the Eighth Heaven from the book, Impressions of Granada and the Alhambra*

Attached to the ceiling panels are over 8000 geometric pieces of different-colored inlaid wood, forming seven successive bands of interwoven star polygon designs. Each level represents one of the seven heavens of the cosmos through which the soul travels, until reaching at the peak, the eighth heaven of the Muslim paradise, represented here as a small cupola filled with *muqarnas*.

Figure 4: *View of the Seven Heavens ceiling*

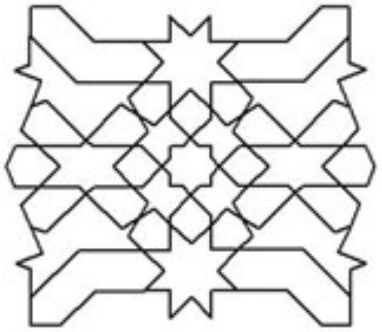
Although faded now, the wood pieces were once painted in glowing colors. Dating from the reign of Muhammad V in the 14th century, the *Seven Heavens* ceiling is a masterpiece of marquetry and served as the prototype for other, subsequent *artesonado* ceilings in Islamic Spain [1].

Geometric Islamic designs in general, and the *Seven Heavens* ceiling in particular, may be recreated using only a compass to draw circles and a straight edge to draw line segments. This paper will illustrate how the diamond-shape motif and the eight different, regular star polygon designs found in the *Seven Heavens* may be constructed in a manner described by El-Said and Parman [3] and also by the author ([4] – [10]), using only the geometer's tools of straightedge and compass (or the electronic equivalent, the *Geometer's Sketchpad* software [11]).

Analysis of the Geometric Structure

If one were to lay flat the four wooden panels consisting of half of the summit and one of the four sides of the *Seven Heavens*, the resulting schematic of the ceiling design would look like Figure 5. below. This figure is a copy of one found in the *Official Guide: The Alhambra and Generalife* [12]. After some reflection, an underlying grid becomes apparent, with two motifs comprising most of the design. The two motifs are a diamond-shape (found in two forms) and a 16-pointed star polygon, shown in Figures 6. and 7., respectively, on the following page. The diamond shape is centered on the most common and basic shape in Islamic designs, an eight-pointed star, known as the *khatem sulemani* (or Solomon's seal) [13]. It may be constructed from two congruent squares, offset by 45 degrees from one another and inscribed within a circle.

Figure 5: *Schematic of half of the summit and one of the four sides of the Seven Heavens, laid flat;* from the Official Guide: The Alhambra and Generalife



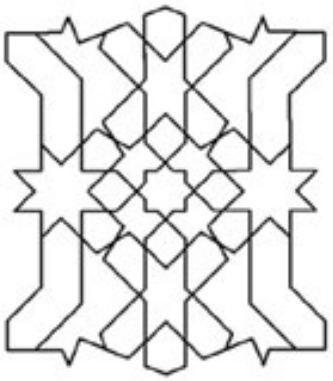


Figure 6: *The two diamond shapes; the second one is offset from the first one by 90 degrees*

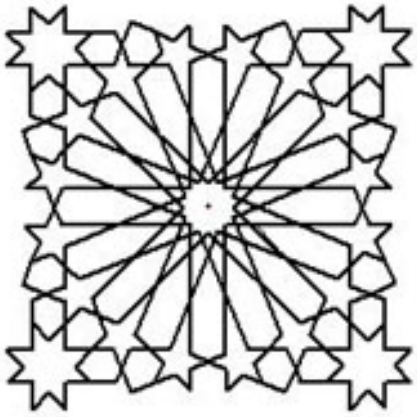


Figure 7: *The 16-pointed star polygon design (Star 2)*

By recreating and connecting the diamond shapes and the 16-pointed star design, a grid for the *Seven Heavens* ceiling starts to take shape, as in Figure 8. shown below. Counting from the bottom up, Levels 1, 3 and 5 contain only these two motifs, which alternate from left to right, starting with all or part of the diamond shape. It is interesting to note that the diamond shapes in Levels 1, 3, 5, 7 and 8 are of the first form in Figure 6. and of the second form (that is, rotated 90 degrees with respect to the first form) in Levels 2, 4 and 6.

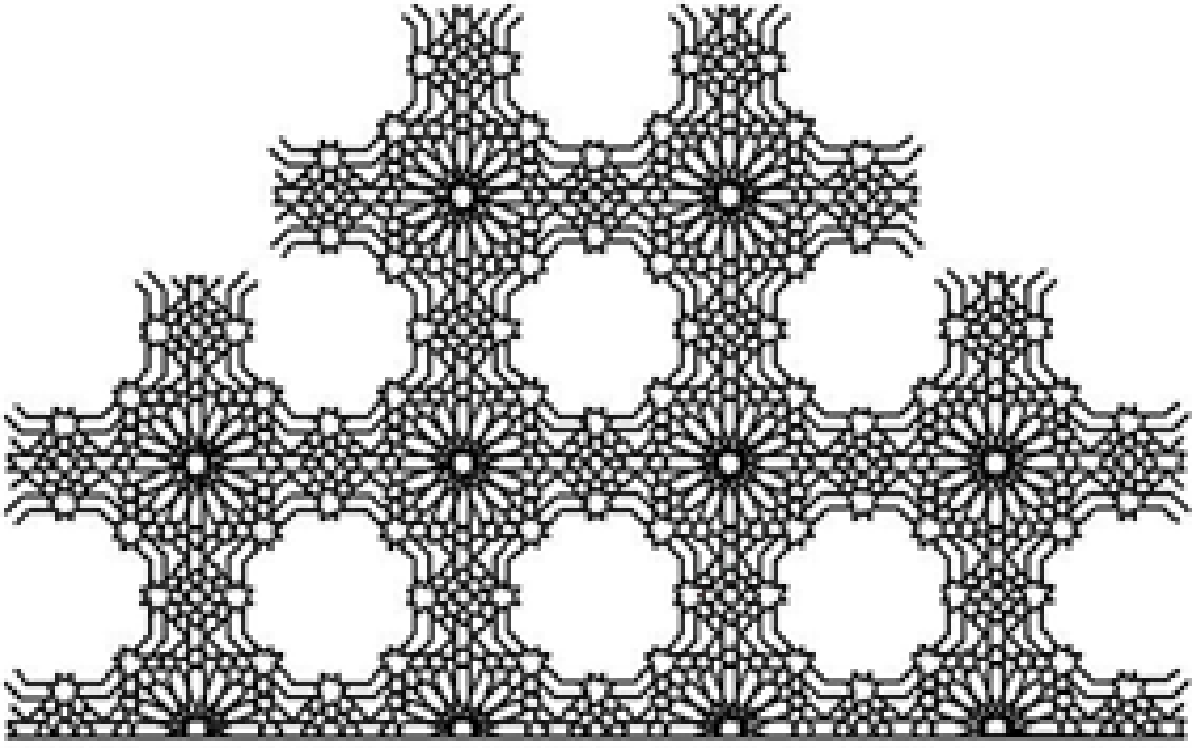


Figure 8: *Grid consisting of only two of the design elements, a diamond shape and a 16-pointed star*

With the formation of a grid of diamond shapes and 16-pointed star polygons, the remaining star polygon designs (shown in Figures 9. – 13.) may now be constructed within each of the apparent regular octagons, or on either side of the diamond shape on Level 7 for Star 8, (see Figure 14.). The star surrounding the cupola (on Level 8.) consists only of an outer band containing eight-pointed stars, which may be created in a manner similar to any of the star polygon designs shown in Figures 9. – 13.

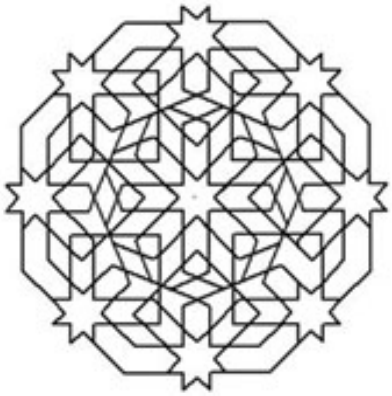


Figure 9: *Star 4 motif found in Levels 2 and 4 (in the center)*

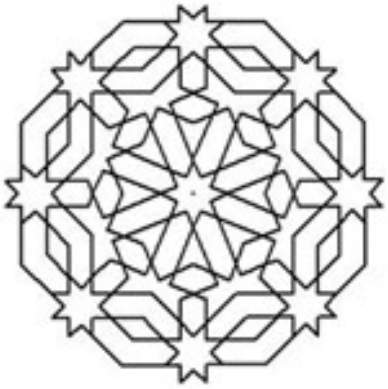


Figure 10: *Star 6 motif found in Level 2*

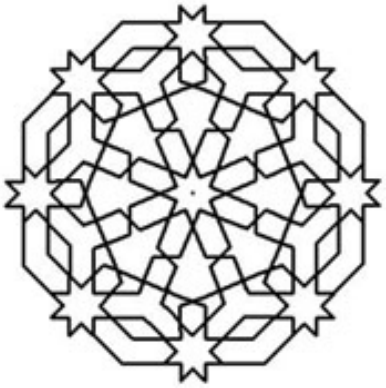


Figure 11: *Star 3 motif found in Level 4*

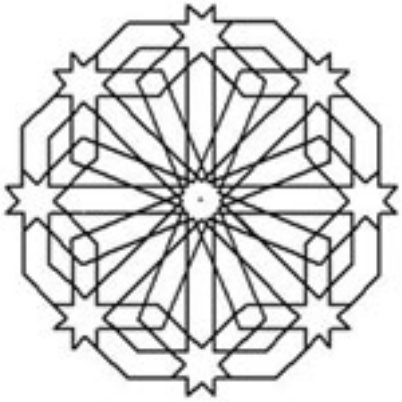


Figure 12: *Star 1 motif found in the center of Level 6*

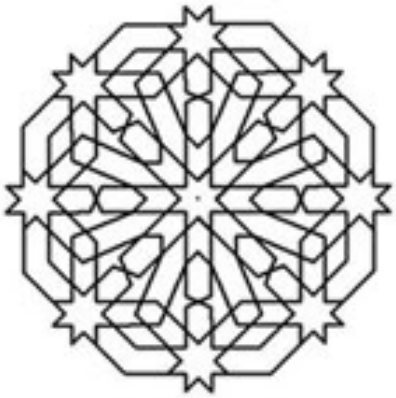


Figure 13: *Star 5 motif found in Level 6*

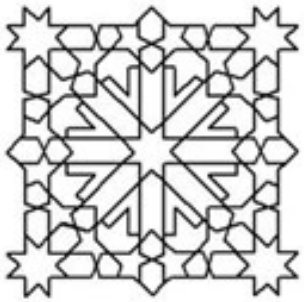


Figure 14: *Star 8 motif found in Level 7*

All of the star designs discussed so far (and shown above) were relatively straightforward to construct from either squares inscribed in circles or line segments joined to points of a regular octagon. The stars within the regular octagons (Stars 1, 3, 4, 5 and 6) were all surrounded by equally-spaced congruent eight-pointed star polygons. The stars contained within square shapes (Stars 2 and 8) also contained the same eight-pointed star polygon at the vertices of each square. The star motifs 4 and 6 (given in Figures 9. and 10., respectively) are found in Level 2, symmetrically arranged on either side of the middle star (Star 7), with Star 4 found on the far extreme ends. The star motifs 4 and 3 (in Figures 9. and 11., respectively) are found in Level 4, with Star 4 found between two copies of Star 3. The star motifs 1 and 5 (given in Figures 12. and 13., respectively) are found in Level 6, with Star 1 sandwiched between two copies of Star 5. Lastly, Star 8 (given in Figure 14.) is found in Level 7. Actually, two halves of Star 8 motif flank a diamond shape, which is itself adjacent to an octagonal ring of eight-pointed stars surrounding the cupola. For the placement of the star polygon designs, please refer to Figure 8. on the preceding page.

The last star to be discussed, designated as Star 7, is the most elaborate one of the entire ceiling and may be found in the center of Level 2. It is quite different from all of the others, and, in fact, the grid in Figure 8. had to be altered to accommodate it (see Figure 15. below). Star 7 does not share any common eight-pointed stars or any other common design motifs with its neighboring star polygons. Even parts of the surrounding diamond shapes were replaced with other shapes in the construction of Star 7.

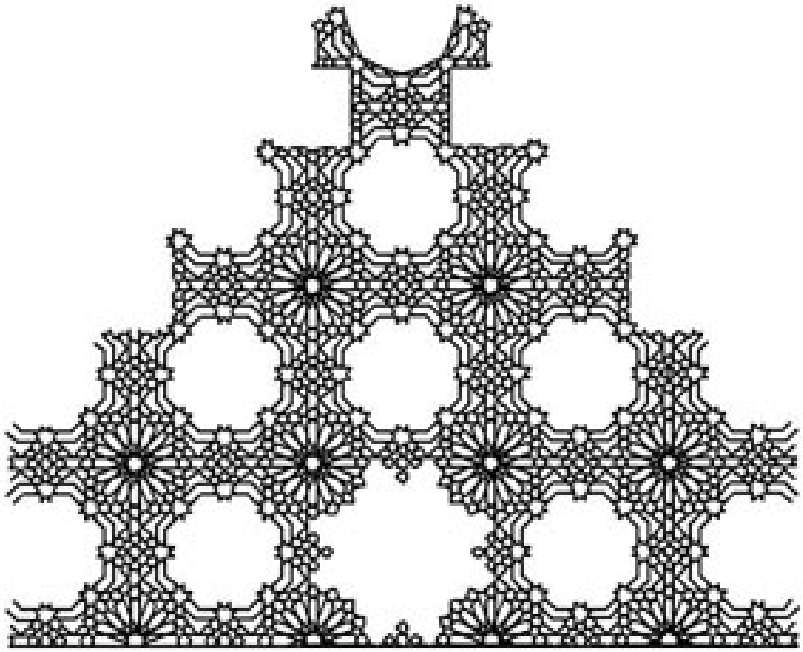


Figure 15: *Grid structure modified to accommodate Star 7 in the middle of Level 2.*

To recreate Star 7, four copies of Star 2, the 16-pointed star motif (see Figure 7. on a preceding page), were created with their centers on the vertices of a square. The four inner eight-pointed stars of Star 2 were removed, and then lines parallel to line segments that already existed were drawn. Connecting the points of intersection generated by these new lines, this process was continued, until all of the design of Star 7 (shown in Figure 16.) eventually emerged.

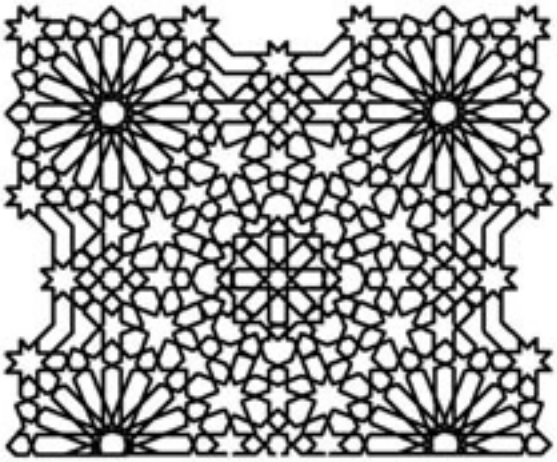


Figure 16: *Star 7 design, found in the middle of Level 2*

Discussion

All of the individual, Islamic star polygon designs discussed here (and recreated in an idealized skeletal form) have either eight or sixteen points and are highly symmetric, with four-fold rotational symmetry (about a central point interior to the eight-pointed star polygon) and four mirror reflections at 45 degrees to one another through that point. As a result, they are all classified as belonging to the *p4m* symmetry group, one of the most common classifications for Islamic patterns [13]. The diamond shape has two-fold rotational symmetry and mirror reflections, and so may be classified as belonging to the *pmm* symmetry group. The overall pattern of the *Seven Heavens* ceiling may not be classified using the 2-dimensional symmetry groups, since it is not all contained in a single plane. None of the three individual planes comprising each of the four sides exhibits any overall symmetry. However, all four sides are identical, and in that regard, the *Seven Heavens* may be considered highly symmetric.

Small eight-pointed star polygons may be found at the centers of the diamond shapes and the seven of the star polygon designs, as well as at the four vertices of the square designs (see Figures 3. and 14.) and the eight vertices of the regular octagon designs (see Figures 9. – 13.). Sixteen-pointed star polygons may be found at the center of Stars 1 and 2. All of the star polygons designs were straightforward compass and straightedge constructions, except for Star 7, whose construction required a surrounding square grid of 16-pointed star polygon designs (centered on the vertices of a square) from which to start.

The medieval Islamic artists and woodworkers conceived and created an extraordinary stepped pyramidal ceiling in the throne-room of the Alhambra. The *Seven Heavens* is very unusual and unique; no other ceiling like this exists in Islamic Spain. Consisting of 8017 pieces of wood, the ceiling is a masterpiece of marquetry, serving as the prototype for all subsequent *artesonado* ceilings. Finding a way to recreate the design was a great and rewarding challenge for the author.

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