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Modular Origami in the Mathematics Classroom

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Abstract

In this paper, the authors describe what modular origami is, make a case for using modular origami in the mathematics classroom, and provide a progression for classroom implementation.

1. Overview

Many papers and discussions have focused on using origami in the mathematics classroom. However, few have mentioned modular origami. Modular origami is similar to origami, except instead of folding one single object, the folder creates many identical objects (units), then pieces the units together to form a much larger shape (such as a Platonic solid). By incorporating the "modular" extension to the origami, the folder transitions into a three dimensional mode of thinking (spatial reasoning), which is a much higher level of thinking. This paper discusses the merits of modular origami in the mathematics classroom as well as provide an implementation plan for the instructor.

2. Why Modular Origami is Important

Modular origami fits into the mathematics classroom quite neatly. One of the major difficulties students face with mathematics is the subject seems to be very procedure driven – such as solving a linear equation. Modular origami can aid students with this struggle. It teaches students how to follow a procedure – repeatedly in fact! The authors note an improvement in their student's ability to follow procedures after completing a few modular origami structures. Modular Origami also teaches students to see relationships between different models by examining various modifications of the procedure, and to work cooperatively [1]. By varying the procedure, the students are able to generate their own unique structures, as well as teach others the new assembly. This is one of the goals of every mathematics classroom – generating unique solutions and sharing it with others. The instructions have become internalized by the student and made their own, thus, enhancing the mathematical power of the students.

Problem solving is another area in which modular origami increases student learning. With the new NCTM standards, mathematics teachers have placed problem solving and open-ended problems into greater focus [2]. Modular origami helps students achieve this. The "fitting together" of the identical pieces into a final shape helps students think in a more spatial and creative way. In this regard, the students become problem solvers – elevating their thinking while creating the shapes. This elevated thinking helps students see problems from a broad range of viewpoints and offers many approaches students can follow in obtaining solutions.

Modular origami provides students mathematical experiences in a very different way. Instead of discussing the properties of squares, parallelograms, rectangles, etc., students create them and explore their relationships. Rather than taping together nets to form platonic solids, the students create the solids through a more unique and eloquent means. In other words, modular origami brings students back to the mathematics of shape. The figures made are inheritably mathematical in both appearance and form.

Modular origami has many other benefits in the mathematics classroom. Through discovery learning, the participants engage in a constructivist, student-centered activity. This is illustrated by observing that the student, rather than the teacher, is producing and assembling the figures. Modular origami transcends age, race, etc. It has been used with children as young as 10 and adults in their 60's, as well as English language Learners (ELL) [3]. For the authors' particular implementation, high school students with no previous knowledge or skill in modular origami were chosen and instructed in a "success/learning for all" approach.

3. Using Modular Origami in the Classroom

3.1 Materials needed. While one can certainly purchase special origami paper, for classroom use we find that to be cost prohibitive. However, more economical alternatives to this special paper are easy to come by. Scrap paper can be saved and cut up into squares of any size for folding. At our school, the print shop saves their scrap paper and uses a guillotine paper cutter (model: challenge 305) to cut the paper into squares. We have also used memo cube paper from various office supply stores. The advantages to the memo cubes are the wide range of colors available as well as accessibility for students to explore further constructions on their own.

3.2 Open-faced Dodecahedron. We begin by assembling an open-faced dodecahedron using Pentagon Module (108 Degrees). See Figure 1. A wonderful description of this can be found at Jim Plank's Penultimate website as well as further extensions and variations. [4]

Our reason for selecting this assembly as the beginning module is because the first three folds provide the base for most modular origami pieces we use. The open architecture also provides the students easier manipulation of the pieces and requires only 30 units. The folds are fairly simple with only the final fold providing a slight difficulty for some students. Having students work in cooperative groups expedites the production of the individual pieces and instills student ownership towards assembly of the polyhedron.

Terminology such as square, rectangle, parallelogram, pentagon, and of course, dodecahedron are incorporated into the lesson. In this way, we are building upon the student's knowledge of polygons and expanding it to include polyhedrons. Another teaching opportunity arises when the students explore the relationship between paper size and the volume of the resulting shape. This concretely helps dissuade students from the common misconception that doubling the length doubles the volume. It also demonstrates to the students' exponents and their geometric relationship to length, area and volume.

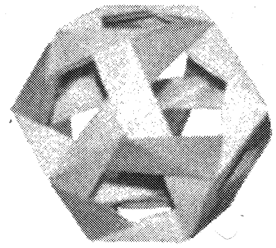

Figure 1: *Open-faced Dodecahedron*

3.3 Mette Unit Wreaths. This construction is intermittently used depending on the age and abilities of the students. Some students who struggle with the spatial assembly of the 30-piece dodecahedron find success with the less taxing wreaths. See Figure 2. Although assembly is often easier, several of the variations are both aesthetically appealing as well as challenging for the students. These variations both increase the complexity of the construction as well as the assembly of the wreaths. Mette Pederson's has published several books which go well beyond the wreaths [5].

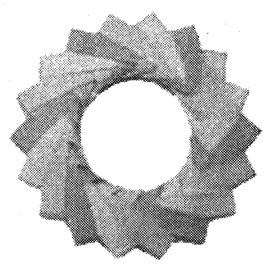

Figure 2: *Mette Unit Wreath*

**3.4 Sonobe` Units** (The author's favorite, as well as many of our students): The versatility of the forms provides students with great fundamentals to extend the origami lessons. Several variations of the Sonobe` exist, but the construction the students have had the most success with is from Helena Verill [6]. While the open-faced dodecahedron requires exactly 30-units assembly, the Sonobe` can be formed from a 3-unit hexahedron to unlimited constructions resembling geodesic domes. The students begin by assembling a 6-unit cube. Once constructed, these pieces can be reconfigured to form a 12-unit stellated octahedron, or the more popular 30-unit stellated icosohedron. See Figure 3.

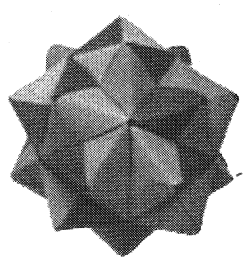

Figure 3: *Stellated Icosohedron (30- unit Sonobe`)*

Several interesting extensions spring forth. One compares the differences between the 30-unit pentagonal module that forms a dodecahedron, and the 30-unit Sonobe` that forms a stellated icosohedron. This comparison helps the students explore how these objects are named, the importance of definitions, and allows them to discover the connections between the names, sides, edges, and vertices. Two other extensions come into play when the assemblies are expanded. The first is the idea of positive and neutral curvature. The intersection of both four and five pieces form a positive curvature, while the intersection of six units has a neutral curvature. From this, students can explore the interplay of combining these intersections to form a myriad of other forms. The most popular is the assembly of 90-units. This interplay of both hexagons and pentagons can best be modeled by a common soccer ball further grounding the activity with common-day objects.

3.5 Other Variations. Besides the variations from Mette unit wreaths, Penaltumate, and Sonobe` students are encouraged to attempt further models from the internet and books. An authors' favorites is

Tomoko Fuse` Unit Origami which contains numerous models using several approaches and techniques [7]. Another type that students show an interest in, is classified as planar, modular origami. These models can best be described as being comprised of intersecting planes in space.

Students also are encouraged to explore these models as a three-dimensional color problem. For example, any of the thirty piece models mentioned earlier can be assembled using chromatic numbers three, five, or even six. This requires increased spatial reasoning and students to postulate as to the number of combinations possible. They then can assemble the models to concretely prove or disprove their postulates introducing such concepts as counter-example, and proof by example.

4. Conclusion

Modular origami supports the mathematical classroom in many different pedagogical ways. It provides opportunities for cooperative groups, establishes unique mathematical experiences, builds connections between mathematics and the arts, teaches procedure, and provides unlimited extensions and permutations using the basic units discussed. For the classroom teacher, the assessment is not difficult. Modular origami is an extendable activity with no deadlines for completion, provides immediate and direct feedback to the student, and is product oriented.

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