

Duality: A Common Thread in Math, Science, Literature, and Art?

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"Symmetry as wide or as narrow as you may define it, is one idea by which man through the ages has

tried to comprehend, and create order, beauty and perfection." - Hermann Weyl

"The only great truths are those for which the opposites are also true." - Niels Bohr

Extended Abstract

According to the Oxford dictionary the word "*dual*" means being twofold. In philosophy there is the theory of "*dualism*" that argues that any domain of reality is always underpinned by two independent, opposing principles, for example, mind and matter, form and content, idealism and materialism, etc. In many of the theologies and religions of the world we also find the pervasive idea that the forces of good and evil are equally balanced in the universe. Another common idea is that of the dual nature of human beings, existing both in body and spirit. In the Christian doctrine there is also the belief in the dual personality of Christ (human and divine).

Traditional Chinese philosophy similarly believes that there is both an active male (*yang*) and passive female (*yin*) principle in the universe, which is beautifully embodied in the symmetric yin-yang as shown in Figure 1.

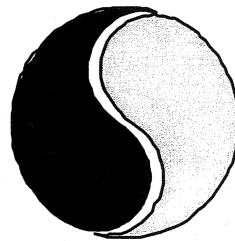


Figure 1: Duality of the yin-yang

Several political theories also show evidence of a kind of dualistic thinking. In Marxism, for example, we find a dialectical view of the relationship between the theory and empirical practice (*praxis*) of society and political systems, the thesis and anti-thesis, a continual tension between capitalism and socialism, as well as between the proletariat and the bourgeoisie, i.e. the so-called "*class struggle*".

Even in everyday language, a kind of duality exists between simple antonyms (opposites) such as hot and cold, tall and short, love and hatred, strong and weak, etc. Basically, the one concept is defined by and understood in terms of the other, and together they form wholes, which complement and enrich each other.

Basically duality is a special kind of symmetry. The word "*symmetry*" immediately evokes the ideas of balance, proportionality, order, harmony, and beauty. Perhaps more precisely defined, two things, objects or ideas may be called dual if the one may be obtained from the other by simply interchanging corresponding dual concepts or properties. If the resultant interchange is the same thing, object or idea, then it is called "*self-dual*".

Apart from the yin-yang symbol, many common, everyday objects are self-dual. Consider for example a chessboard and how interchanging the black and white squares as shown in Figure 2 simply produces another chessboard (albeit rotated by 90°).

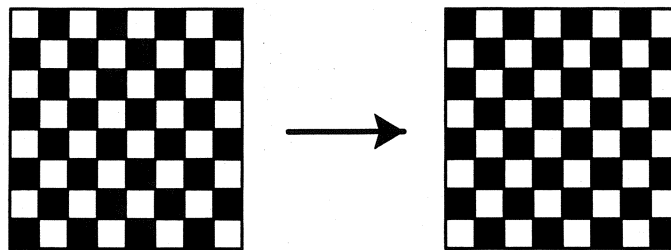


Figure 2: Duality of Chessboard

This paper intends to present some elementary examples of a few beautiful dualities within the following contexts, some of which are perhaps less well-known:

- art - one or two examples of Escher's work [1]
- science - the wave-particle duality of light, electron spin, etc.
- mathematics - side-angle duality in plane geometry [2], Fibonacci series-product duality [3], $\sin(x)$ - $\cos(x)$ duality in trigonometry
- literature - one or two examples of duality in my own poetry and prose [4]

References

- [1] D. Schattschneider, *M.C. Escher: Visions of Symmetry*. W.H Freeman & Co., NY, 1990.
- [2] M. de Villiers, *Some Adventures in Euclidean Geometry*. Univ. Durban-Westville, 1996.
- [3] M. de Villiers, A Fibonacci generalisation and its dual, *IJMEST*, 31(3), pp. 447-477, 2000.
- [4] M. de Villiers, *Unquenchable Thirst*, Lux Verbi, Cape Town, 1988.