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Great Kiva Design in Chaco Canyon: An Archaeology of Geometry

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Abstract

Principles of classical geometry are applied to kiva architecture in Chaco Canyon. Structural features of these circular buildings suggested that the prehistoric Anasazi may have known how to construct regular polygons. Both structural and quantitative aspects of kiva architecture are examined. Concentric rings of Chaco Canyon's kivas were measured to detect presence or absence of proportional signatures of known geometric designs. A result of this examination suggests that great kiva architects were familiar with a technique known as squaring a circle.

I. Introduction

Geometric themes often pervade prehistoric American architecture and iconographic styles. This penchant for geometric themes is particularly true in the desert Southwest where a puebloan culture called Anasazi once dwelled in the Four Corners region. The Anasazi were a prehistoric culture, ca. 400-1300-AD, with a sophisticated agriculture and astronomy, extensive trade networks, and complex architectural and engineering practices. Chaco Canyon, New Mexico was their ceremonial center. Among the pueblos and great houses, sunken multi-ringed structures called kivas often rest within square perimeters among rectangular residences (Figure 1). [1]

This article discusses the application of ancient (or classical) geometry as a tool to investigate the architecture of Anasazi kivas in Chaco Canyon. [2] The Anasazi disappeared around 1300 AD. No European ever saw an Anasazi, much less spoke to an Anasazi architect. All we really know about them is what we find buried in the ground, in the remains of cliff dwellings, or painted on walls. Ancient geometry provides a level of objectivity, both as a design methodology and as source of quantitative data. Given the geometric nature of the Anasazi remains, both architectural and iconographic, it made sense to base an investigation on geometric patterns and designs, and their proportional constants.

II. Design

For mathematician Jay Kappraff, there are general elements that typify design:

> "All good design should have:
> 1. Repetition - some patterns should repeat continuously.
> 2. Harmony - parts should fit together.
> 3. Variety - it should be non-monotonous (not completely predictable).
> Many architects and artists would add to this a fourth requirement that the proportions of a design should relate to human scale." [3]

With respect to archaeological approaches to design, two leading scholars on Chacoan architecture, John Stein and Steve Lekson, share a concern for recovering the general meaning of the built-form phenomena found in the canyon. "What are the basic parts of the architectural composition and what are the critical issues that organize them?"

> If we are ever to understand Chaco, we must first endeavor to understand the set of "rules" (syntax) that structure the basic architectural vocabulary of the Anasazi-built environment. This is design. A dictionary definition of design is 'the arrangement of parts of something according to a plan.' In contemporary architectural design, building morphology (the design solution) is a mosaic of issues synthesized into the whole."[4]

They argue for architectural specialists charged with designing, supervising and sponsoring the efforts. "'Image' is a critical function of ritual architecture, and is attained through special techniques that collectively compose a 'sacred technology'." What this "sacred technology" is, however, remains undefined.

The design solution suggested here focuses on the simple geometry used for constructing the regular polygons. Figure 3a summarizes the types of operations that generate the regular polygons. The first clues that this type of geometry may have been operating among the Anasazi architects were the pilaster arrangements in some of the smaller "clan" kivas. When present in the Canyon, these features divide the kiva's circle into either six, sometimes eight, and occasionally ten equal parts (Figure 2d,e,f). With a peg and cord, one can accurately and quickly make all the points necessary from which to base further complex designs. The triad of the triangle, square and pentagon, along with their truncated derivatives and internal proportions relative to the original radius provide a dynamic framework by which to investigate and illustrate potential design traditions employed by the Anasazi kiva builders. Architectural precision is key to the kind of information this system can generate. In this regard, the masoned ruins of Chaco Canyon have more integrity than their adobe counterparts. Given Chaco's cultural primacy, there is reason to assume that whatever was done there was done to "code." The regular divisions of the pilasters, and for the niches of great kivas have proven to be fairly exact [5].

III. Great Kivas: Organization of Great Kiva Floor Features

The interplay of circle and square is implicit in the kivas surrounded by quad roomblocks. Circle and squares enjoy a couple patterns common to architectural design. One is related to the v2 progression of squares, a pattern previously identified among southwestern tribes, prehistoric and historic (Figure 4) [6]. Figure 5 superimposes a circumscribed square constructed from the floor (inner) circle. Numerous variations on a theme are possible, and two are shown. Both account for the posthole placements that supported huge beams that carried the roof. The perimeter of the suite of floor features were all contained in the floor circle's inscribed square. This was a consistent pattern for all of Chaco's great kivas. One of the ramifications of the v2 model is that the distance between adjacent postholes and the radius of the floor circle of great kivas *should* be equivalent, a feature that could be tested in the field.

IV. Squaring a Kiva?

The four-petal pattern within a circle Figure 5b is implied in the construction of the pentagon-decagon (Figure 3f,g). It is also significant because: it forms a nucleus of an operation that can "square a circle." Squaring a circle means that a circle's circumference and area matches the perimeter and area of a given square (Figure 6). When superimposed on the kiva illustrations, the intersections of the larger circles seemed to fall onto the outermost rings of the great kivas (Figure 7).

If this was intentional, it is a provocative finding that would address the presence of quadrilateral structures and circular kivas, namely, the symbolic resolution of circle and square, heaven and earth, within the structure of the great kiva. A mathematical implication of this occurrence is that the radius of the floor circle and the radius of the kiva's outer wall should approximate a 1: 1.272..., the square root of phi (1.618...). Phi is an important ratio that turns up in many natural structures [7] To explore the possibility that v phi was actually present in great kiva ring sets, it was necessary to consult previous field data and garner the radii comprising the ring sets of each kiva in order to evaluate the designs quantitatively.

V. Kiva Radii

The focus of the ring data study was to calculate intra-ring ratios of individual kivas to assess the proportional relationships between individual rings and quantified as ratios. If ratios corresponded to proportional constants, it could imply design structures hidden within the rings. This was carried out by dividing the floor radius into the radii of the larger concentric rings. Assuming a peg and stretched cord technology, each radius represents a fossilized length determined by the kiva's builder.

Table 1 illustrates the results of the exercise. Radii of the floor circles, benches and outer wall perimeter were garnered or deduced from previous reports [8]. The ratios for the inner floor to outer wall radii all reflected a vphi relationship.

Table 1a. Great Kivas Ring Radii (in feet)

	Floor (FR)	Bench	Inner Wall	Outer Wall	Alcove
Court Kiva 3, C K*	14.3	-	16.69	18.21	20.36**
GK B, Pueblo Bonito	20.0	22.38	25.51	-	
GK A, Pueblo Bonito	22.5	24.35	25.72	28.73	
GK Chetro Ketl	25.98	27'-29'*	30.62	33.3	
GK Rinconada	27.64	29.37	31.75	35.12	

Table 1b. Great Kiva Ring Proportions

Great Kivas	Bench/FR	Inner Wall/FR	Outer Wall/FR	Alcove's Arc/FR
Court K 3, CK	-	1.167	1.273	1.423
GK B, PB ***	-	1.118	1.275	
GK A, PB	1.08	1.14	1.276	
GK Chetro Ketl	-	1.178	1.281	
GK Rinconada	1.06	1.147	1.27	

* CK - Chetro Ketl
**Arc of alcove in concentric relation to common center

*** PB - Pueblo Bonito

VI. Discussion

The models for reconstructed pilaster arrangements provided theoretical license to apply proportional laws to great kiva designs. The spatial models are consistent with actual lengths and proportions. All models were well within the technological capabilities of the Anasazi, especially in view of the expert engineering skills required to build the kivas and multistory pueblos. Given the continuities of method, statistical ratios and spatial organization provided by the v2 and vphi ring models, it may indicate that this type of approach is in the right ballpark. The shared ramifications of these models, such as the way they play out on the floors of great kivas, would also support the idea. However, given the potential variety of techniques that can result in the same proportions, we may never know exactly which technique was used nor the order of construction of the component parts. Was the set of floor features built before the concentric walls? Or vice-versa? At this initial stage of investigation, given the overlapping and shared ramifications of the two ring models, it is preferable to have multiple design solutions than none at all. At this stage, the models would appear to demonstrate an architectural vocabulary, syntax and general continuity that bring together the parts of great kivas in a cohesive way. The design elements — pilasters, rings, postholes — were not realized the same way in every case. Variations between great kivas are quite obvious to the eye. None are exactly the same. This demonstration of applied ancient geometry accounts for many of the variations. If it is true that most, if not all, of the variations are derived from the "same page," it is also true that we have yet to understand how big that page actually is.

In general, a new class of data is provided by this simple yet rigid and replicable geometric method of investigation. It is simple because of its connect-the-dot format, yet rigidly grounded in the laws of proportion. A working knowledge of this tradition can isolate and deliver an entirely new series of concrete, quantifiable phenomena to prehistoric architectural studies. A unification of spatial constructs with their mathematical ramifications expands the types of questions that can be asked.

• Is the radius of the inner circle the initial radius selected by the kiva architect? If not, what was the sequence of ring construction? Was this sequence always the same?

• Does a vphi ring signature represent a metaphorical intent to reconcile the opposing or contrasting characteristics of circle and square?

• What would be the value of constructing a data base that tracks radii and kiva styles over time and space within and between Anasazi sub-regions?

These questions have yet to be resolved, but without this type of methodological framework they may never have been asked. Ancient geometry provides a way to penetrate questions of this order because it reveals potential cognitive frameworks in a behavioral, measurable context. All of the spatial data garnered by this method is subject to quantitative evaluation by other investigators without recourse to highly specialized and intricate statistical operations [9].

The geometry used to explore the kivas of Chaco Canyon is a simple set of techniques with elegant effects. It also reflects a framework not of our own making, but one that has been repeatedly discovered, recovered, developed and diffused throughout the Old World. Was the geometry discovered and developed independently in the New World? The Chaco Canyon kiva designs suggest that it probably was. Earlier

Hopewellian earthworks [10] and the circular temple complexes of Western Mexico (Figure 8) [11] display a similar geometry on much larger scales. It could be a major tradition running rampant throughout Prehistoric America. It could be restricted to a few isolated regions. Identifying and detailing the extent of this spatial language will be a fruitful archaeological challenge to our powers of observation, recognition, and prediction.

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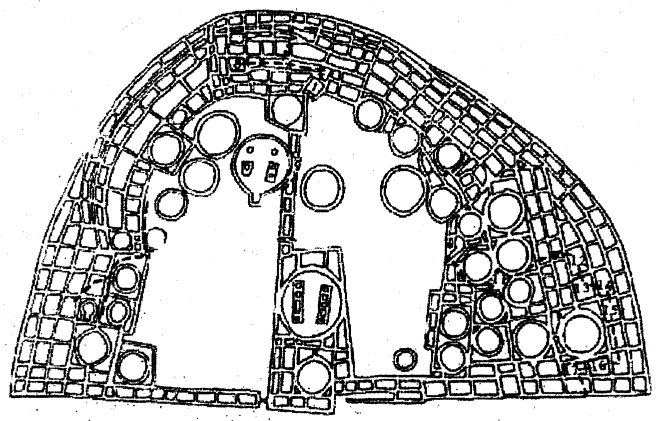
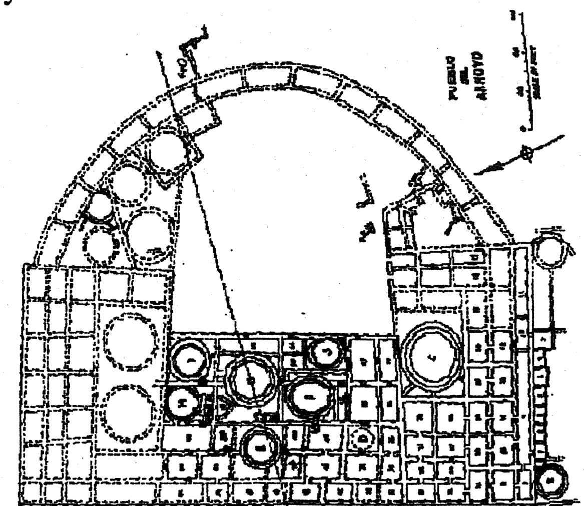
5 Pages of Figures.

Figure 1. Chaco Canyon Pueblos



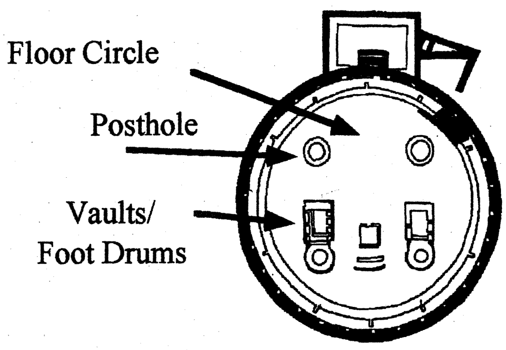
a. Pueblo Bonito (Judd 1964)
Oldest and Largest Pueblo in the Canyon.



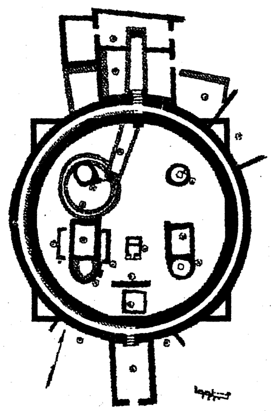
b. Pueblo del Arroyo: Planview (Judd 1959)
Quad Rooms and Circular Kivas

Figure 2. Chaco Canyon Kivas

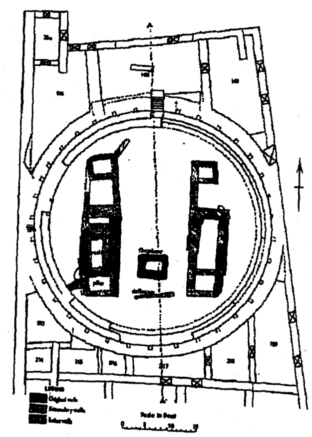
Great Kivas



a. Great Kiva
Chetro Ketl

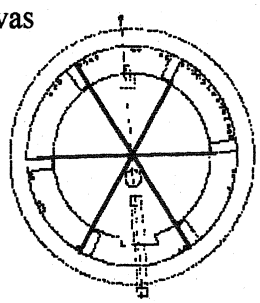


b. Great Kiva
Rinconada

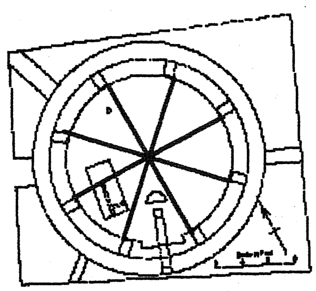


c. Great Kiva A
Pueblo Bonito

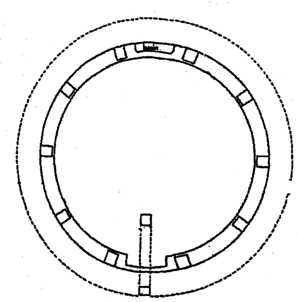
"Clan" Kivas



d. Kiva R
Pueblo Bonito
6 Pilasters: Hexagon



e. Kiva C
Pueblo del Arroyo
8 Pilasters: Octagon



f. Kiva D
Pueblo Bonito
10 Pilasters:Decagon

Figure 3. Regular polygons constructed from a circle. Same techniques may have been used at Chaco Canyon for "clan" kiva divisions.,

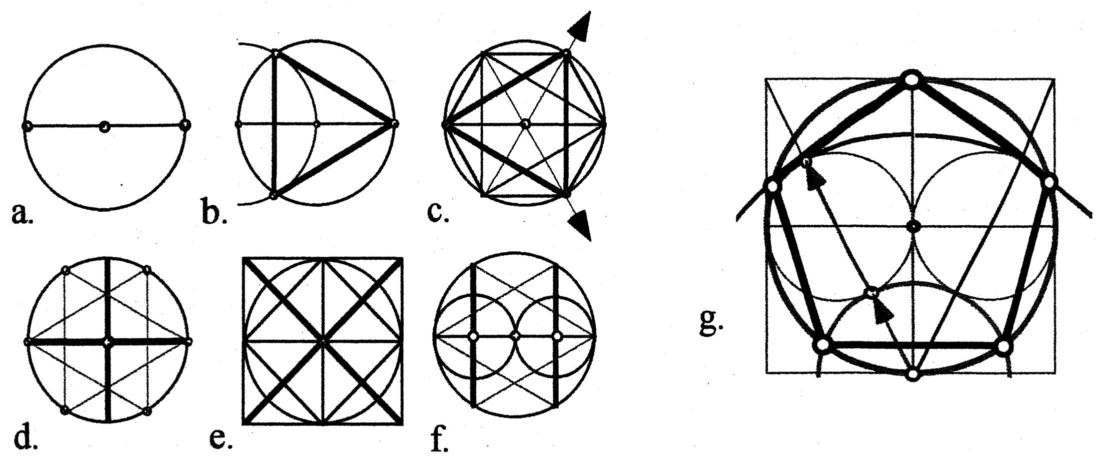


Figure 4. Common √2 designs and patterns from Southwest Pueblo societies.(from Zaslow & Dittert 1977, Fig. 8)

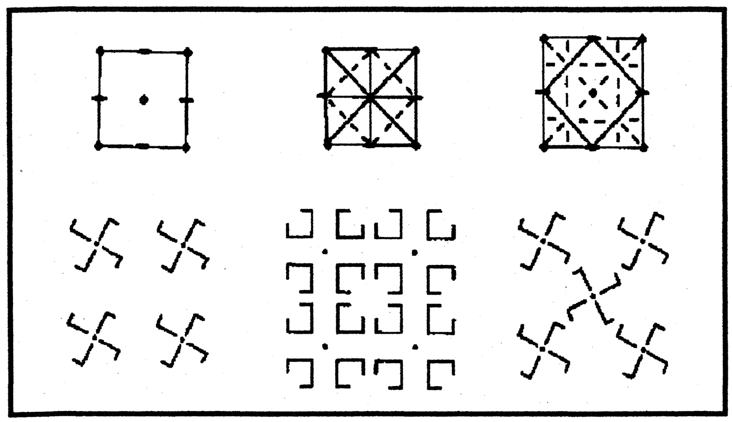


Figure 5. A couple of √2 models superimposed on the floor features of the great kiva of Chetro Ketl in Chaco Canyon. Alternating square-circle constructs account for the entire perimeter of the suite of floor features and posthole placements.

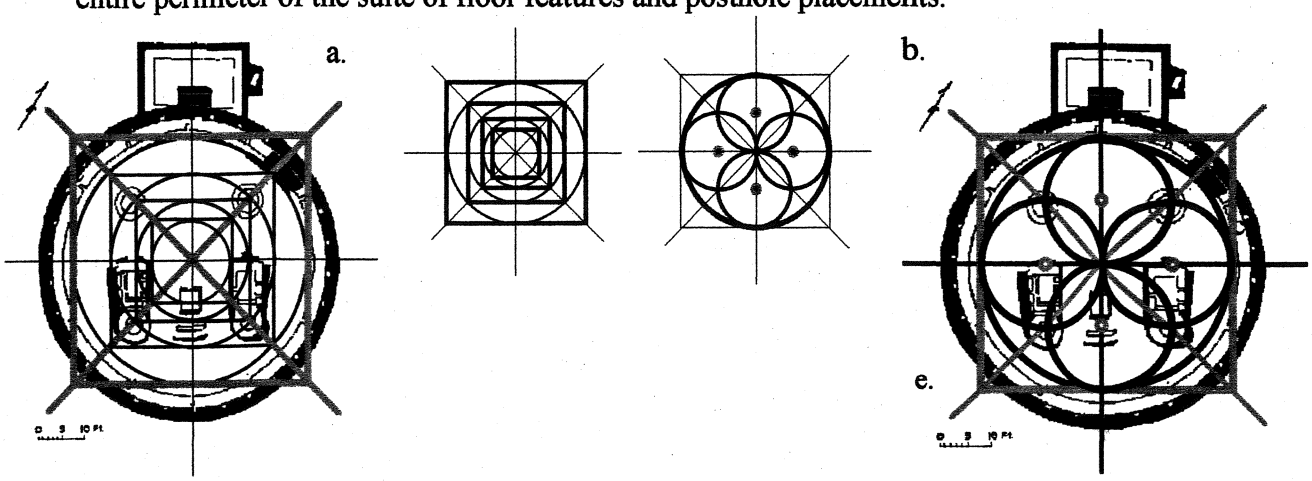


Figure 6. A Technique to Square a Circle. Working from a base associated with the construction of a pentagon-decagon, intersections outside the initial circle for the radii for a concentric circle. The derived circle's circumference is virtually the same as the perimeter of the floor's outer square.

rCircle a = 10m
$2\pi r$ = circumference (C)
C = 10m x 2π = 62.83...m
Outer Square Perimeter = 80m.

Radius Circle Y= radius **ab** x ϕ (phi)
1.618... = phi; 10m = 16.18...m.

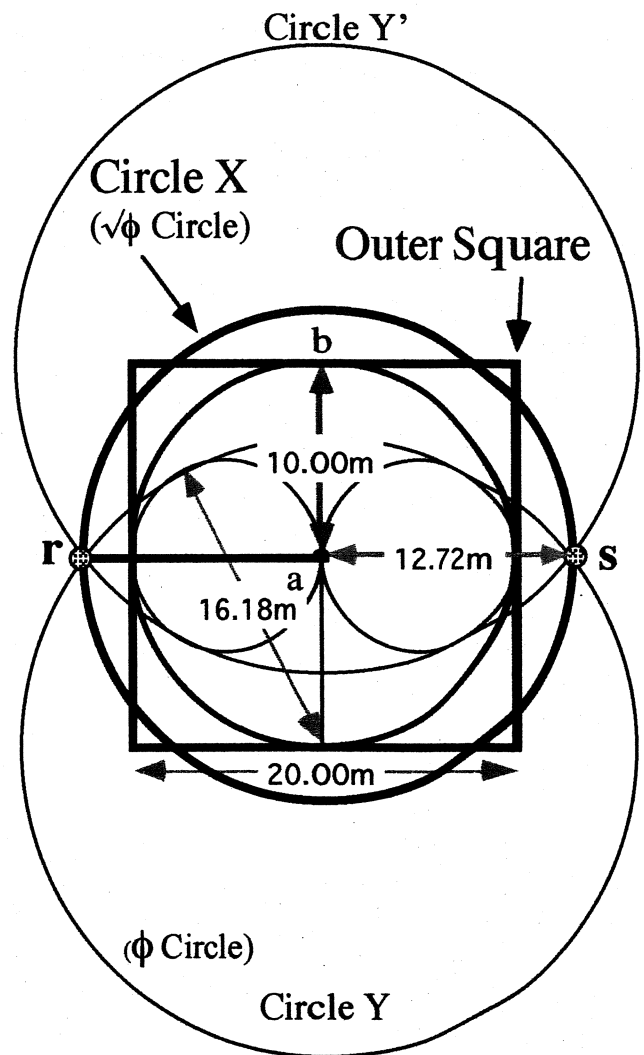
Circles Y and Y' intersect at r and s.

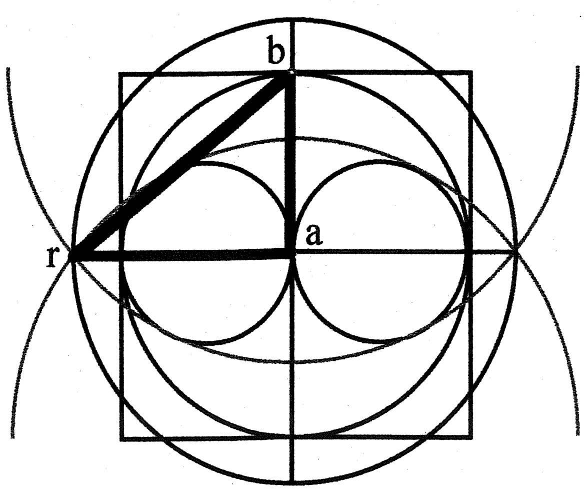
rCircle X / rCircle a = $\sqrt{\phi}$ = 12.72...m.

$2\pi r$ = circumference
Radius ar = $\sqrt{\phi}$
Circle X circumference:
$2\pi(\sqrt{\phi})$ = 79.92...m
Outer Square (80m) ≈ Circle X (79.92m)

Proof: Pythagorean Theorem
Δabr
ab = 1
ar = $\sqrt{\phi}$
br = ϕ

(after Lawlor 1995: 74-76)





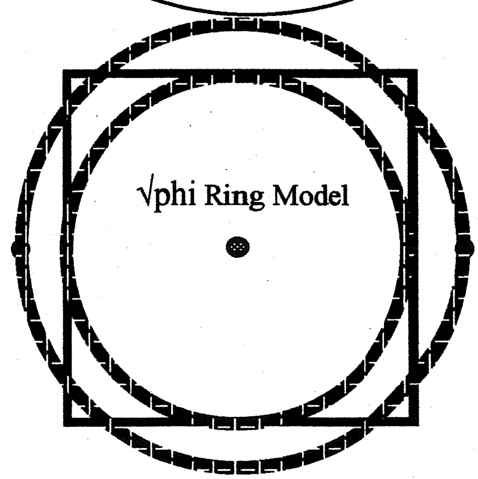
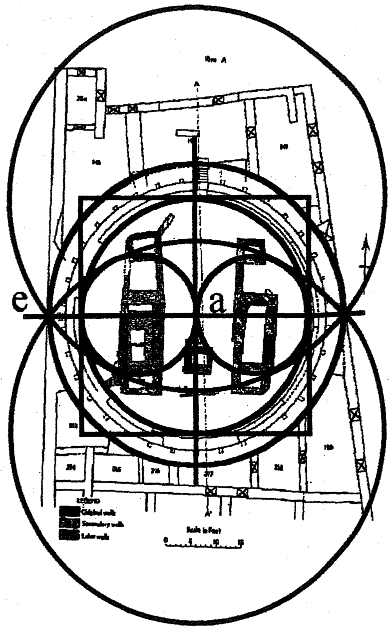
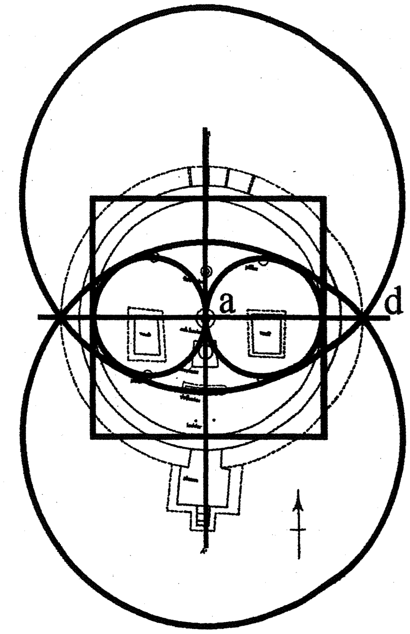


Figure 7. √phi Ring Model and Great Kivas.



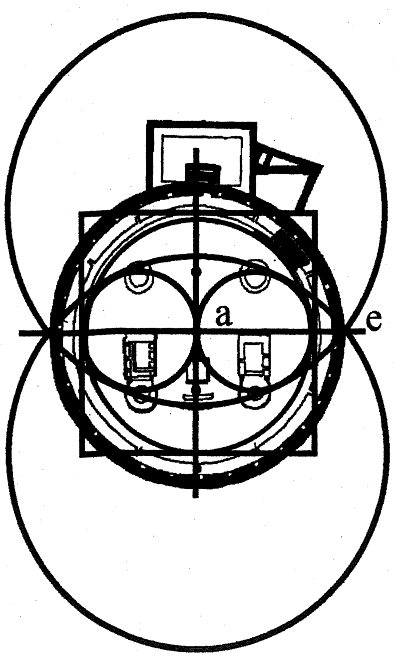
Great Kiva A
Pueblo Bonito

Radii:
ab: 22.50
ac: 24.35
ad: 25.72
ae: 28.73
ac/ab ≈ 1.08
ad/ab = 1.14
ae/ab = 1:276 ≈ √phi



Great Kiva B
Pueblo Bonito

Radii:
ab: 20
ac: 22.38
ad: 25.51
ac/ab = 1.118
ad/ab = 1.275 ≈ √phi

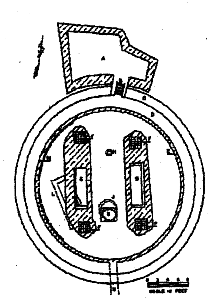


Great Kiva
Chetro Ketl

Radii:
ab: 25.98
ac: 27'-29'*
ad: 30.62
ae: 33.30
ac/ab = ≈ 1.05-1.11*
ad/ab = 1.178
ae/ab = 1.281 ≈ √phi
*2nd ring is uneven.

Court Kiva 3, Chetro Ketl
Combination √phi and √2 ring set.
The court kiva's curving alcove requires a radius that matches a √2 ring model.

ad/ab = 1.273 ≈ √phi (99%)
ae/ab = 1.423 ≈ √2 (99%)
ac/ab = 1.167
Radii:
ab: 14.30
ac: 16.69
ad: 18.21
ae: 20.36



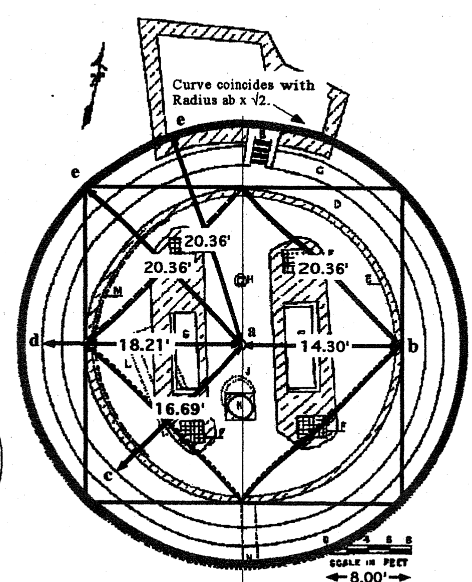
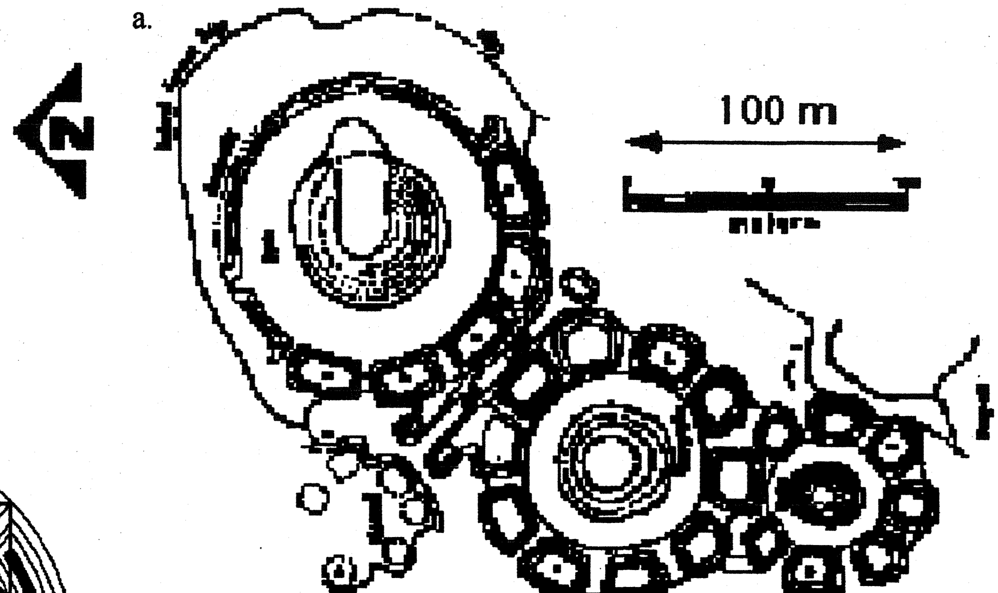


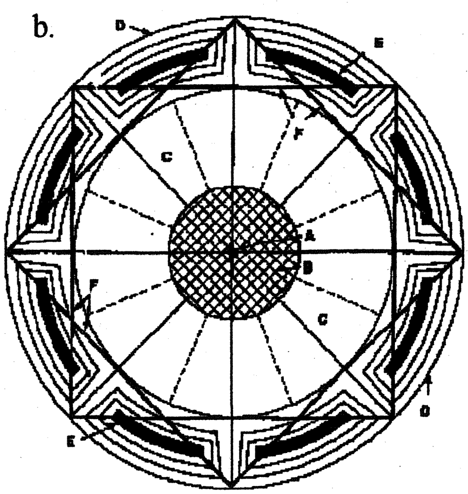
Figure 8. Circular Temple Architecture From Western Mesoamerica: 200 B.C. - 500 A.D. An Origin of Anasazi Circular Architecture? To pursue the roots of geometric traditions in the New World, ancient geometry provides a framework for common and repeating patterns that can function as clues as we proceed deeper into the prehistoric past.

a. Guachimonton precinct at Teuchitlan, Jalisco (after Weigand 1996, Figure 4). The southern and central circular temple complexes are divided into 8 and 10 sections, respectitvely. They are connected by a shared platform. The central complex is separated from the largest by a ball court. Given the apparent correlation between the number of divisions and the diameter of the circles, it would appear that the largest circle may have been envisioned with twelve divisions.

a.



b.



Formal Design Elements of Western Mesoamerican Architecture (as described in Weigand 1995)

A. radical center of the family of three concentric circles.
B. 1st circle of the family, the pyramid
C. 2nd circle of the family, the patio
D. 3rd circle of the family, the banquette's outer wall
E. platforms/pyramids atop the banquette circle
F. dependent geometric square

b. A primary component of the circular temple complexes of West Mexico is a relatively common $\sqrt{2}$ design. Weigand's "dependent geometric squares" are a product of generating the overlapping squares of an octagon. In turn, intersections generated by this overlap provide the points necessary for dividing the circular structure in sixteenths.