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Mathematical Connections
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The "Mathematics and Culture" Project

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Abstract

It is no great surprise that mathematical structures and ideas, conceived by human beings, can be applied extremely effectively to what we call the "real" world. We need only to think of physics, astronomy, meteorology, telecommunications, biology, cryptography, and medicine. But that's not all – mathematics has always had strong links with music, literature, architecture, arts, philosophy, and more recently with theatre and cinema

1. Introduction: the aim

Morris Kline wrote in his famous book: "Mathematics in Western Culture": [1]
"It is less widely known that mathematics has determined the direction and the content of much philosophic thought, has destroyed and rebuilt religious doctrines, has supplied substance to economic and political theories, has fashioned major painting, musical architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe."

It is no great surprise that mathematical structures and ideas, conceived by human beings, can be applied extremely effectively to what we call the "real" world. We need only to think of physics, astronomy, meteorology, telecommunications, biology, cryptography, and medicine. But that's not all – mathematics has always had strong links with music, literature, architecture, arts, philosophy, and more recently with theatre and cinema.

The "Mathematics and Culture" project has developed over the years. In the mid-1970s, a project began regarding in particular the links between mathematics, art and images: the *Mathematics and Art Project*. [2, 3, 4, 5] The *Mathematics and Art Project* started in 1976. The project was to make films/videos in which to compare the same theme from a mathematical and an artistic point of view, consulting both mathematicians and artists. Not just filming long discussions between artists and scientists on the theme of the connections between art and science – too vague – but a real confrontation on the visual ideas of artists and mathematicians. "To make the invisible visible," as the artist David Brisson says in the film *Dimensions* [6] made in 1984 with Thomas Banchoff.

So the general plan of the project was almost clear – to make films on the visual relationships of the forms created by artists and mathematicians. The themes of the first two films were soap bubbles and topology, in particular the Moebius band. To have more visual ideas and objects to film, we eventually decided to include the connections between mathematics and architecture, as well as all the other sciences, in particular biology and physics, without excluding literature and even poetry. And, why not, also cinema. From the very beginning of the project there was the idea of focusing on the cultural aspect of mathematics, the influence and the connections of mathematics and culture, of course starting from the point of view that mathematics has always played a relevant role in culture, being an important part of it. All this was carried out using the most potent visual tool: filming. [7]

One of the most important subjects was of course the Dutch graphic artist Maurits Cornelis Escher. The first conference was held in 1985 [8], the second in 1998. [9] Moreover, a film was made with H.S.M. Coxeter and R. Penrose on the subject of Escher's works. [10] All these activities involved students from universities and high schools. We also organised a number of seminars, workshops, courses, and discussions on graduate dissertations, both with the students and with the teachers.

As these were the general lines of the project, it was quite natural to consider as a part of it the organization of exhibitions (several were organized in the following years), congresses and seminars, the publishing of books (well illustrated!), even for students of mathematics, the history of art, and architecture. We were also determined not to ignore the educational aspects of the project that involved mathematicians and non-mathematicians from several different countries.

In the late 1990s, it seemed that the time had come to begin another project that would be integrated into the previous one and which would extend it. In 1997, the series of congresses on mathematics and culture began. Since the first event in 1997, the conferences have been organized on themes: mathematics and art, mathematics and cinema, mathematics and history, mathematics and economy, museum and exhibitions of mathematics, mathematics and literature, mathematics and technology, mathematics and medicine, etc.. In particular, one session in March 2002 was devoted to Mathematics, Tibet and Ancient China. [11]

Now the goal of the series of events (and the publication of the proceedings by Springer every year) [12] is to construct a sort of database of the relationships between mathematics and culture, looking to modern and ancient times.

In this paper, I would like to give an idea of how all these ideas are presented at the annual Venice conferences, giving examples from the more recent events. Showing the aim of constructing a permanent bridge between mathematics and culture, or even better, to show very clearly that mathematics is a relevant part of culture, so no bridge is needed.

2. Mathematics on stage

The International Mathematical Union, announcing that 2000 was the International Year of Mathematics, on May 6th 1992 in Rio de Janeiro, set itself three principal aims. First of all "the great challenges of the twenty-first century"; just as, at the World Conference in Paris in 1900, David Hilbert had listed a series of great problems that the mathematicians would have had to face during the course of the twentieth century, one of the aims of the International Year of Mathematics is to focus the attention of mathematicians on the great challenges for the new century. Another aim of the Year: the key for development. Pure Mathematics and Applied Mathematics are the most important keys for development. Which means a great effort towards scientific education, especially in those countries in which access to scientific knowledge is difficult.

A further aim is to re-launch "the image of mathematics". In our modern society of information, in which scientific knowledge plays an increasingly important role, mathematicians and mathematics, however, do not seem to exist, they do not seem to be present. Mathematics and mathematicians are practically never mentioned by the media.

On November 11th 1997, the general conference of UNESCO approved resolution 29C/DR126 and decided to sponsor the International Year of Mathematics. It is no surprise that in recent years mathematicians have figured large in the world of show business – in movies and theatre, as well as in books. Books dealing with mathematicians have had enormous success all over the world – books such as Simon Singh's "Fermat's Last Theorem" [13], "The Number Devil: a Mathematical Adventure" by Hans Magnus Enzensberger [14], "Uncle Petros and the Goldbach Conjecture" by Apostolos Doxiadis [15]. Not to mention successful plays about mathematicians in the theatre. Starting with "Arcadia" [16] by Tom Stoppard, Oscar-winner for the screenplay for "Shakespeare in love". Stoppard, who has a passion for physics and mathematics, made a videocassette with the mathematician Robert Osserman on behalf of MSRI (Mathematical Science Research Institute), Berkeley, California. [17],[18] In the video, Osserman and Stoppard discuss the mathematical aspects of "Arcadia" while actors perform the scenes on another part of the stage. Obviously Stoppard said he didn't know anything about the mathematics referred to in his play, and that he was only interested in whether the plot worked for theatre or movie performance. However, there's no doubt that he took the trouble to find out something about the subject. In "Arcadia", Stoppard imagines the story of a self-taught mathematical prodigy. The 13-year-old heroine, Thomasina Coverly, discovers the set which subsequently came to be known as the *Mandelbrotset* — as well as intuiting the first inklings of fractals. All this in 1809, many years ahead of time. Obviously, instead of the *Mandelbrotset*, it is called the *Coverly set*.

But the real boom in plays dealing with mathematicians took place in 2000 and 2001. In 2000 (perhaps because it was World Mathematics Year?) there were several plays being performed at the same time in New York, on Broadway or at off-Broadway theatres, dealing with mathematicians. The New York Times on June 2, 2000, had a two-page article in the theatre supplement entitled "Science Finding a Home On-stage". The writer of the article Bruce Webern made the forecast that one of the off-Broadway shows, called *Proof*, would be very successful. And that is what has happened.

Proof by David Auburn [19] opened in late May 2000 at the Manhattan Theatre Club. Theme: the world of mathematics. The "proof" referred to in the title concerns a problem of number theory, but the author never gets to the bottom of the mathematical problem (why should one?). Auburn says that his play doesn't attempt to "prove theorems". But the encounter with mathematicians, rather than furnishing specific mathematical information, helped the author and the actors to realise that mathematics is not an arid subject; mathematicians enjoy themselves, they discuss, argue, they get excited. "It was a surprise for all of us." Auburn also confessed that he didn't do very well in math at school. He says that today we live in a technological age, in which technology itself produces a host of "dramas". Maybe the "two cultures" division is breaking down.

The play was so successful that, from October 2000 it moved to a large Broadway theatre, and dates for the USA tour have been fixed right through 2002. *Proof* has also been officially recognised. It won three Tony Awards for best play, best actress (Mary-Louise Parker), and best director (Daniel Sullivan). In addition, the play won the Pulitzer Prize for theatre in 2001.

The play has been staged in many other countries over the last two years, always with success.

Another extraordinary success was staged at the "Piccolo Theatre" (European Theatre of Milan) in March 2002, then in Spain. The show will be repeated in 2003, and there will also be a version in Arabic. The play called "Infinities," produced by Luca Ronconi, is based on a text by John Barrow [20], and it puts mathematics itself on the stage. There are no characters, there's no plot – the leading role is played by mathematics. How can one put "science on the stage"?

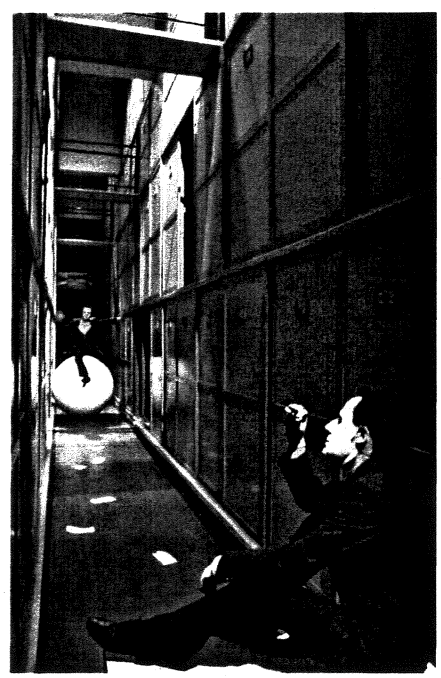

Figure. 1
Infinities, Courtesy of Piccolo Teatro © Piccolo Teatro Milano 2002.

Luca Ronconi has written: [21] "I feel that – as was shown some years ago in Italy, by writers such as Vittorini and Calvino in the field of literature, not to mention an engineer like Gadda – in the age of science we live in, in a period when more than ever before our daily lives are being affected one way or another by the results of scientific thinking, I feel that science might prove to be the best stage for a dramatic event that is truly contemporary.

In order for the language of science to develop all of its revolutionary and innovative power in the theatre, I feel that it needs to be transcribed faithfully for the stage, avoiding any attempt at explanation. In other words, to design a truly "scientific" theatrical event (not merely one based on a scientific subject), I'm convinced that one has to forego the politically correct approach of divulgation, and that instead one has to aim to show the exquisitely esoteric nature of the rarefied fields of today's science."

Very clear words uttered a year before the show went on stage, although the project was clearly already underway. So when Ronconi wrote those words, he already had in mind what he wanted to do. The show deals with the paradoxes of infinity. In his presentation for the play "Infinities", Ronconi clearly says that he didn't want to produce a show based on divulgation or demonstration. Rather, it was more like an exhibition; one goes to theatre not to understand, but "only" to have the chance to take in, to grasp some idea, but mainly to be involved in the theatrical event. Mathematics as emotion, one might say.

Reviewers have said that the show is continually evolving, and that spectators (if they felt like it) might see it again and again ad infinitum without leaving the theatre (save when the performers collapse from exhaustion). This is Galileo's idea that we can't appreciate the infinity of the universe with our finite mind, but only a small part of it.

When I went to see play, my first impression was that the mathematical part functions perfectly well. The first station – Hilbert's infinite hotel – that infinite upward space, with the numbers, the people, the floors, the mathematics, with those figures that appear on the liquid crystal screen – all this makes it fascinating to watch the actors the first time you see the play; and then, what happens when you come back after the end (?) of the last station. And those figures on the screen - do they explain anything? I would say not. Signs, images, suggestions, irony (one of the characters says that there is no simpler explanation). And the clever thing is that all of us – director, spectators, actors – are experimenting, we are in a space that certainly isn't infinite but which is very similar to what we imagine an infinite space to be like. We test our reactions, we try to find answers, and those signs help us, they suggest solutions, maybe they confuse us. Are we involved with science? Those signs on the screen remind us that we have to find a way of explaining the problem in clear, simple words. So, our science will be useful. If you like, Ronconi's work is a play based on language. One might say that this is obvious – every play is based on language. But Ronconi's idea was to experiment with a new language (for the theatre), that of mathematics, and to build around this "esoteric" language (as he calls it) a theatrical performance which for this reason has to develop a new stage approach. It's a totally new experience for the actors, for the audience and for the director.

I have had the chance to experiment, to compare, to understand what it means to be a theatre director, and to appreciate what human creativity consists of. In different places, with different cultures and languages, where the unifying theme – mathematics – was an ideal bridge because of its main characteristic – its universality.

4. Mathematics and films

The most interesting development in the last dew years has been the great interest that the cinema has aroused through several festivals involving films linked to mathematics (at the "Piccolo Theatre" - European Theatre of Milan, at the Venice film library, at Bologna university, at an arts cinema in Rome); several conferences have also been held on the same subject [8]. It is interesting to note that all these events were sponsored by departments of mathematics. In particular, those in Rome by the Institute for Higher Mathematics and by the Department of Mathematics "G. Castelnuovo", University of Rome "La Sapienza", showing the major efforts that mathematical institutions are making to arouse their students' interest.

Obviously, the film that attracted most attention was "A Beautiful Mind". In December 2001, Ron Howard's film "A Beautiful Mind" [22] was released in the USA. It is the story of the mathematician John Nash who won the Nobel Prize for economics. He is the mathematician responsible for the well-known theorem of regularity by De Giorgi-Nash, a famous example of a theorem proved by two mathematicians using different techniques, working separately and unbeknown to one another, but in the same period.

This example is referred to by the Italian mathematician De Giorgi, who has a page in the book on Nash, and in the video interview I made with him in 1996. In March 24, 2002, the film received four Oscars for best film, best director, best non-original script. Crowe playing the part of a mathematician is a very significant sign. [23] At the Venice congress in 2002, Harold Kuhn, Princeton University, scientific consultant for the movie, explained what sort of interaction took place between the director, the cast, Russel Crowe in particular, and the mathematicians. [24]

The story of John Nash tells of the solitude of a mathematical genius who, for most of his life, was cut off from the world around him – a world with which he was unable to communicate. It must have been a real challenge for an actor at the height of his career to take on a role dealing with madness and mathematics in the same film. One might be tempted to say that film-makers are only interested in mathematicians who are mad or who at any rate meet a tragic end (for instance, "*Death of a Neapolitan Mathematician*" by Mario Martone), but this is not so.

In 1998 the Oscar for best screenplay was awarded to Matt Damon and Ben Affleck (another story about mathematicians) for the film *"Good Will Hunting*" by Gus van Sant [25].

The film is based on the book with the same title by Sylvia Nasar [26].. The film does not refer to Hilbert's nineteenth problem, or to the De Giorgi-Nash theorem. The film does not go into Hilbert's problem because it is not easy to explain. Of course, the problem is basically one of language. But the whole question also concerns madness which is just as difficult to describe and understand. All in all, I find that the film is mathematically correct and fascinating, in the sense that one is attracted to the story and the characters. I say this both as a mathematician who is a spectator, and as a spectator who is also a mathematician.

In 2002 alone, two more films on mathematical subjects were released: "Enigma" [27] and "The Bank".[28]

On September 4, 1939, the mathematician Alan Turing arrived at Bletchley Park, about fifty miles north of London, to take part in a very important project. Bletchley Park was the headquarters of the GCCS (Government Code and Cipher School). At the outbreak of World War Two, the British secret services, the famous SIS or M16, ran the centre for deciphering enemy communication codes. At the time, Turing was the leading expert on computation. His ideas about computable numbers and his device called "Turing's machine" eventually led to the development of the first computers.

The main problem for the secret services at that time was deciphering the code system used by the Nazis. And the codes were mainly based on their Enigma machine. And they were unbreakable codes, according to the Germans.

On May 7, 1941, an Enigma machine with all its instructions was captured, and for the first time it became possible to decode messages almost in real-time, so that immediate action could be taken. By August 1941, British intelligence was able to decode any message in less than 36 hours. However, the "Foreign" code system used by German vessels far from their homeports was never deciphered.

On February 1, 1942, all German submarines changed their code system. The British decoding devices were no longer any use. Work had to start again from square one. Once the Americans entered the war, their secret services began building their own machines, much faster and in larger quantities than the British could, since they had more funds available. By the end of 1943, the position of even the most distant U-Boat was known; not even the captains knew their position with the same precision as the Allies.

But the Allies avoided taking too much action based on the decoded messages, so as not to raise suspicion, letting the enemy know that they had the key to the coding system. Speed was of course essential in decoding messages in order to be able to take advantage of the information. In December 1943, the first fully electronic machine, named Colossus, came into operation. This story, for reasons of national security, was only made public thirty years later and formed the basis for Robert Harris's book "*Enigma*".

Tom Stoppard, playwright and director, also wrote the script for the film entitled "*Enigma*" directed by Michael Apted, and produced by Mick Jagger. In the film, no mention is made of Turing. Instead there is a fictitious mathematician, Tom Jericho (the book does mention Turing in passing) who has the same motivation as Hardy. He's a mathematician who falls head over heels in love (this seems to be the fate of mathematicians in films). Although he's good-looking, he dresses sloppily and is something of a dreamer. He has great intuitions, and is something of a genius. It's a very English film, slow-moving, accurate, which draws the characters and the situations well. We meet mathematicians, we see where they work, though we don't exactly know what they do; they're called swots but they're likeable. It's a film that reveals itself little by little. Stoppard has completely changed the original ending as in the book, and his new version is better.

Turing committed suicide in 1954; in 1953, he had been convicted of homosexuality and forced to undergo a hormone cure.

The other film had a mathematician in the leading role and dealt with a subject that is much in vogue – the chaos theory and Mandelbrot's fractal geometry. Distributed by "Axiom Films" (could it have been called otherwise?), this Australian film was directed by Robert Connolly, his first feature film as screenwriter and director. The title is "*The Bank*". The lead part is a mathematician Jim Doyle (played by David Wenham). His antagonist is the CEO of the bank – beautifully played by a sinister Anthony Lapaglia who has just the right face for the part. The film is also a thriller – there is a subplot (much more conventional than the main one) running alongside the duel between the mathematician and the banker.

Doyle is a clever mathematician (it's worth noting that almost all films involving mathematicians depict them as clever, whether they are also crazy, schizophrenic, or ruthless murderers). His idea is to use the chaos theory, in particular the search for attractors and fixed points, to cause a slump on the world's stock exchanges, thus enabling the bad banker to earn billions. The love-hate relationship between the two characters is the best part of the film, even though the baddie steals the stage in most of the scenes.

The film is full of interesting information and is sprinkled with shots of Mandelbrot's sets (incidentally, when the French-Polish Benoit Mandelbrot first saw these patterns, he thought his computer had broken down). And there's also Julia who's not only a fine actress but also has a lot of visual appeal.

One thing that had to be included was a mathematical proof done on a paper tablecloth (yes, because there's a type of cheap restaurant which appeals to mathematicians and is just the place for writing proofs on the paper tablecloth!). Doyle feels strongly that mathematics is his life, and in order to convince the bank's Board of Directors that his method will work, he exclaims "It's mathematics, there's no risk!"

An intelligent and well-made film that captures and keeps your attention. What's more, it doesn't only deal with mathematics, but with a much vaster subject that is almost taboo – the enormous power that banks wield.

For all the other films dealing with mathematics in the last twenty years, including the very interesting film by Simon Singh "Fermat's Last Theorem", I refer to my long paper on "Mathematics and Cinema". [8]

4. The Visual Mind

An important part of the "Mathematics and Culture" project is devoted to the visual aspect of mathematics; a book with the title "The Visual Mind 2" is being published now by MIT Press, following the publication of "The Visual Mind" MIT Press, in 1993. [3], [4] Over the last few years, after a long period of oblivion, artists have begun to take a new interest in mathematics and mathematicians. The talent and creativity of mathematicians, assisted by graphic tools unimaginable until a few years ago, have opened new fields to mathematical research and have given the chance to capture the enormous graphic complexity in very simple problems and formulas. Impossible to imagine, until a few years ago, a book like "Symmetry in Chaos: a Search for Pattern in Mathematics, Art and Nature". The authors, the mathematicians Michael Field and Martin Golubitsky, say in the introduction [29]:

"In our mathematics research, we study how symmetry and dynamics coexist. This study has led to the pictures of symmetric chaos that we present throughout this book. Indeed, we have two purposes in writing this book: to present these pictures and to present the ideas of symmetry and chaos – as they are used by mathematicians – that are needed to understand how these pictures are formed... One of our goals for this book is to present the pictures of symmetric chaos because we find them beautiful, but we also want to present the ideas that are needed to produce these computer generated pictures."

Who could have imagined a few years ago that such declarations could have been found in the introduction to a volume written by two mathematicians? It is possible to distinguish between two types of images: those resulting from the solution of interesting mathematical problems; in this case mathematicians are able to give a precise explanation both of the scientific problem and of the method by which the pictures have been produced. Other pictures are obtained by the visualisation of known phenomena or of algorithms, numerical methods, mappings for which the scientific problem is not so clear, in which it is not possible to show the solution. In the latter cases, pictures are in general less important from the mathematical point of view, but from the figurative point of view all kind of images can be of interest to artists, regardless of their scientific interest.

Mathematicians tend to give priority to pictures resulting from a well-defined scientific problem, while for artists a combinatorial or casual method of variation of the various graphic elements of a scientific picture can be more interesting.

Probably the creativity and capability of mathematicians have put artists in the position of comparing their ideas with those of mathematicians. "And since mathematics possesses these fundamental elements and puts them into meaningful relationships, it follows that such facts can be represented or transformed into images... which have an unquestionably aesthetic effect." This was written in 1949 by the famous artist Max Bill, who died in 1994 [3].

But even more interesting, the new electronic images created by mathematicians have generated new works of art which use more traditional tools. A very interesting example is a recent work of Heleman Ferguson. A few words on the mathematical background. Until 1982, mathematicians knew only three minimal surfaces of the class called minimal – completely immersed surfaces, meaning that the surfaces extend to infinity and never self-intersect. These three surfaces are the plane, the catenary and the helicoid, and a portion of all three can be obtained with soap films. None of the three has a handle; more precisely, their topological type is zero. For almost two hundred years, mathematicians have wondered whether there exist minimal complete immersed surfaces with at least one handle; that is, whose topological type is greater that zero.

Two American mathematicians, David Hoffman and William Meeks III, using the equations found by the Brazilian mathematician Costa, were able to demonstrate the existence of a class of minimal surfaces whose topological type is fairly high: minimal surfaces with holes. The two mathematicians have used graphic techniques to prove the result; it was one of the first interesting examples of a theorem proved using graphic techniques. The shapes of the images subsequently obtained by Hoffman and Meeks aroused interest extending beyond the purely mathematical. Hoffmann himself remarked to an interviewer that the co-operation between art and science produced something significant in both areas. Several artists were fascinated by the forms of the newly discovered minimal surfaces and they used them as models to create sculptures using different materials.

Amongst others, Helaman Ferguson had the idea of applying his technique of computer-assisted sculpting to the Costa surfaces [31]. Using his computer-assisted technology, Ferguson (with the help of mathematician Alfred Gray) created several aesthetic objects. In particular, for the Maryland Science Center exhibition, "Beyond Numbers", a ten-foot diameter Costa minimal surface. Ferguson has described the technique to obtain a bronze, aluminium, fibreglass and Carrara marble Costa surface [32].

Described in art language, Ferguson's sculptures are abstract images but in spite of his use of exacting mathematical equations and content, his sculptures are not rigid geometric configurations nor are they mathematical illustrations. The artist chooses to use these mathematical ideas to create objects that are warm and humanistic. It is not a unique example of interest of an artist for modern mathematics; this new relationship between artists and mathematicians has been very profound in recent years. See for example the catalogue of the exhibition "The Eye of Horus: Art and Mathematics" [33].

5. Some Comments

There is considerable interest in the series of conferences held in Venice every year, attended by about three hundred university and high school students. And the same happens at the film festival in Bologna where the students (and others) wait in line for two hours before each showing, for the 500 seats available – and many of them aren't able to get in.

All these initiatives are conceived to arouse interest in mathematics by highlighting the many links it has with culture in the widest sense. One of the problems of mathematics is that sometimes it risks being self-centred, losing contact with all the other aspects of human culture, and closing itself up in a sort of ghetto which is difficult to get out of. This also leads to loss of contact between students and teachers. The basic idea behind the projects "Mathematics and Art" and "Mathematics and Culture" is to move in exactly the opposite direction. Let people understand that mathematics plays a major role in culture, art, philosophy, literature, the cinema, the theatre, and in the new technologies. In a way, the idea is to move in the direction of a mathematical education that incorporates many of the basic ideas that are at the root of mathematical knowledge. Through conferences, books, exhibitions, films, and with the collaboration of mathematicians from all over the world, it has been possible to build up a vast database of mathematical culture over the last few years. This major long-term project is open to anyone who has the ability and the desire to learn more about the cultural aspects of mathematics. But it is also a short-term investment given that many of the initiatives have already become educational programmes, and that the projects have already been used in schools, universities, and other cultural institutions. The goal of these projects is to provide the basic knowledge for a more effective distribution of mathematical education by supplying the knowledge and the tools to better understand the deep links between mathematics and culture.

What all this means is that we are investing in culture which is the prerequisite for any type of didactic or training activity. The aim is to provide the ideas and the stimuli that will then be worked on and modified by the various users in different contexts. The books in the series "Mathematics and Culture" are also becoming useful tools for acquiring information, suggestions, ideas, bibliographic references, thereby creating interest and stimulation. And the projects do not only concern European culture; gradually they are expanding to include the main civilisations in other parts of the world, to counteract the idea that we Europeans have of being at the centre of the world, the only source of true culture. (see http://www.mat.uniroma1.it/venezia2003)

I'd like to end with something that the famous German mathematician David Hilbert wrote in 1928: "Mathematics knows no races... since it, and the whole world of culture, constitutes a single country."

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