Mathematical Bridges to Philosophy and Theology

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Abstract

Mathematics has a long history of building bridges to other disciplines, from its practical use in the sciences to more aesthetic connections with the arts. There is also a long tradition of bridges between mathematics and the realm of philosophical and theological discourse. In this paper, we take a brief tour of these latter connections. We consider in more detail some philosophical implications in the field of Dynamical Systems.

1. Introduction

Mathematics can be defined not only as the study of patterns and numbers but also as the study of the structure, justification, categorization, and uses of formal reasoning. Mathematicians find beauty in the elegant structure of mathematical proofs. Just as patterns lend beauty to the visual, musical, poetical, and design arts in which they are found, the structure of mathematical thinking can be found applied throughout history in various corners of the humanities, lending its clarity of argument and elegance of presentation to a sometimes unexpected context. One such connection, more natural with its common concern with formal reasoning, has been with philosophy. In the sections to follow, we present a brief survey of bridges between mathematical thought and philosophical and theological questions, from the Pythagoreans through the Ontological Argument of Descartes to contemporary interpretations of chaos theory.

2. The Early Years

There is a long tradition of the application of mathematical arguments to questions of philosophy and theology. As a part of a scientific view of reality, mathematics goes back to Thales of Miletus and the founding of natural philosophy. Although known in mathematical circles today for important contributions to early geometry, the revolutionary contribution of Thales was the notion that multitudes of phenomena could be explained by a small number of hypotheses: he essentially proposed the marriage of physical observation with deductive logical reasoning for explanations of the ways things work [26].

Pythagoras of Samos, and the Pythagoreans that took his name, linked order, number, and divine intellect. The claim of Thales that "all is water" was a key step away from the mythological toward the mathematical; the Pythagorean claim that "all is number" moved beyond rationalization to quantification. This thread of tying together questions and explanations for both how and why things work in the world persists through Socrates and the Seven Sages. Some scholars have posited that either the Pythagoreans were exposed to an early oral tradition of gematria – the assignment of mystical meaning to words based on the numerical values of their composite letters – or the numerological elements that began to appear in Hebrew texts during the intertestamental period followed from the influence of the Pythagoreans [34].

Plato was one of the most significant ancient Greek philosophers in the western intellectual tradition. Among his many contributions is the view of an idealized world of abstract mathematical objects that survives unscathed in the contemporary world of the philosophy of mathematics. The allegory of the divided line in book six of the *Republic* presents the universe as consisting of two realms, one of appearances and one of eternal, abstract forms. The world of appearances constantly changes, while the world of forms is always static. Mathematical objects and God are similar in their unchanging perfection; to know Truth, one must know mathematics.

3. The Middle Ages

St. Augustine was not only a rigorous defender of the Christian faith but also of intellectual inquiry. He wrote at length of mathematical truth and mathematical reasoning; for Augustine, that which is eternal and true can be used to illuminate the faith [2]. Augustine can be said to have "assimilated aspects of the Platonic view of divinity, namely the doctrine of the immutability of number and truth, into Christian theology" [34]. His argument for the existence of God combined a hierarchy of existence with mathematical truths that hold independent of human existence to conclude that there is a God above us in the hierarchy to provide for truths greater than human reason. Augustine's fusion of faith and understanding into a neo-Platonist theology became the dominant Christian philosophy for several centuries. The spirit of Augustine's use of mathematical reasoning to prove the existence of God runs through the works of St. Thomas Aquinas, Descartes, Pascal, Leibniz, Spinoza, Berkeley, Hume, and Kant, centuries later.

One celebrated example of such mathematical reasoning is that of St. Anselm of Canterbury, the outstanding Christian philosopher and theologian of the eleventh century. Anselm presents in the sixty-five chapters of the *Monologion* and more concisely in the single chapter two of the *Proslogion* his celebrated "ontological argument" for the existence of God. His argument proceeds as follows. God is "that than which nothing greater can be thought." If God so defined exists in the understanding but not in reality, then something even greater could be thought to exist: such a being that exists in reality as well as in understanding. This is a contradiction. Therefore, God exists. This turns the classic Platonic argument on its head: rather than existence in the realm of abstract forms implying perfection, for Anselm, that which is perfect must exist.

4. Renaissance and Transition to the Modern

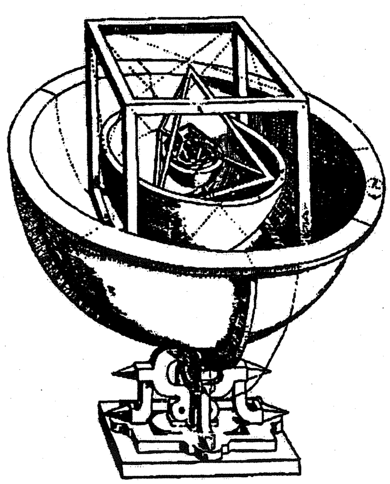

Figure 1: *Kepler's spheres.*

Johannes Kepler is best known for his three laws of planetary motion, which not only linked observational astronomy and mathematics but demonstrated his strong belief that God had created the universe according to a mathematical plan, in the Platonic and Pythagorean tradition. He systematized and extended what was known about polyhedra, and yet he associated each of the Platonic regular polyhedra with one of the classical elements of earth, air, fire, water, and ether. His linkages between the mathematical and the mystical, between mathematical law and the harmony of nature, climaxed in the *Mysterium Cosmographicum*, in which he proposed that the distance relationships between the six planets known at the time could be understood in terms of the five Platonic solids. See Figures 1 and 2. Some ideas die hard; in a recent book, the author "unfolds the astonishing discovery of a complex order to our solar system and puts the ancient idea of 'harmony of the spheres' in an excitingly new perspective" [37].

Two of the best known mathematical arguments concerning the existence of God are from two mathematical giants of the time: Blaise Pascal and René Descartes. Pascal's Wager, as the argument in the *Pensées* has come to be known, parallels Pascal's development of probabilistic theory, essentially using the concept of expected value. The argument goes as follows. God either exists or does not exist, and one can either wager for or against God's existence. Payoffs are finite for each outcome except for the case of a wager for God coupled with the existence of God, which has an infinite payoff. Rationality requires that one assign some positive probability to the existence of God and also that one perform the act of maximum expected utility. Therefore, a rational person must wager for God. Of course, this is an argument for *belief* in the existence of God, not an argument for the existence of God.

The main statement of what has come to be called Descartes' Ontological Argument appears in the Fifth Meditation. This argument is directly logical rather than comparative or computational as Pascal's; it goes as follows. Whatever is clearly and distinctly perceived to be contained in the idea of something is true of that thing. Necessary existence is contained in the idea of God as clearly and distinctly as properties of shapes and numbers that are proven mathematically. Therefore, God exists.

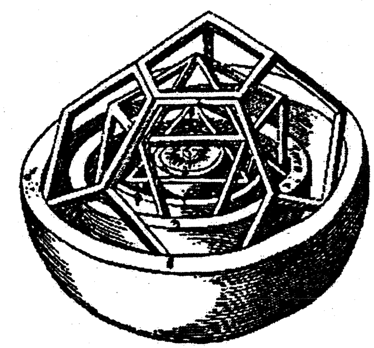

Figure 2: *Kepler's inner spheres.*

Beyond the idea of proof, it could be said that Descartes had a vision of knowledge "as a system of interconnected truths that can ultimately be abstracted into mathematics" [38]. His view of the universe was that of a complex mechanism designed by a Clock-maker God. Humans could understand all of the workings of a deterministic universe if they could discover all of the rules of the machinery. This deterministic view has driven scientific inquiry for centuries, with and without inclusion of the divine architect and rule-maker.

The analytic geometry of Descartes led directly to the calculus of Newton, who sits at the pinnacle of mathematical theology. For Newton, his remarkable contributions to mathematics and physics were tools to "read the mind of God," to provide a better understanding than ever before of the workings of the well-ordered universe of the clock-maker God. The ironic twist is that Newton's greater understanding of the mechanism led those who followed later to a mechanistic philosophy that omitted God. For Newton, the well-ordered universe followed from the perfection of God. Reminiscent of Wittgenstein's image of throwing away a ladder once one has used it to scale a height, Newton's universe was sufficiently well-ordered to be understood without invoking God.

5. An Example from Contemporary Mathematics: Chaos Theory

Bridges between the mathematical and philosophical realms continue to be built in modern times. Some more recent examples include logical positivism, probabilistic causation, and philosophical implications of statistical mechanics. In particular, modal logic has been invoked to revive the ontological argument.

One area of contemporary mathematics that has lent itself to an exploration of philosophical implications is Dynamical Systems. As the name suggests, Dynamical Systems is the study of the properties of mathematical models of systems undergoing change. Its questions are more qualitative than quantitative. Its objects of study are typically either systems of iterated functions or differential equations, dating to Newton's invention of calculus and equations of motion. Its tools range from traditional techniques of classical analysis to various branches of topology born in the twentieth century at least partially in response to some Dynamical Systems questions. Its applications range from ecology to meteorology, from chemical kinetics to population genetics, from economics to celestial mechanics.

A dynamical system consists of two ingredients: a rule, in the form of a function to iterate or a system of differential equations, which specifies how the system evolves with time; and the space on which the rule acts. The basic lines of inquiry are suggested by the following questions:

What happens in the long run for a particular initial condition, or starting value, in the space?
How does "what happens" depend on the starting value?
Do certain points in the system remain fixed? Exhibit periodic behavior?
Do certain points approach a limit, in finite time or asymptotically?
Do different starting values have qualitatively different long-term behaviors?
How does "what happens" depend on the function or differential equation?
How does "what happens" depend on parameters within the function or differential equations?

The questions have a clear potential for translation into philosophical ones: if humans, human social structures, and human physical contexts are points in the universe as a complex dynamical system, what are the ultimate fates of the elements of the space and of the system as a whole? French mathematician Henri Poincaré is generally acknowledged as the founder of the field of Dynamical Systems with his 1889-1890 work on a question that both is typical of the field and has philosophical overtones: is the solar system stable?

One area within the field of Dynamical Systems that has both generated vigorous research activity and crossed over into popular culture is chaos theory. It is an area in which the answers to the questions above differ from what previous approaches, and mathematical intuition, would have suggested. At first glance, "chaos theory" may seem to be an oxymoron: "systematically organized knowledge" applied to "total disorder or confusion." In fact, chaos theory provides a framework for a better understanding of some complex irregular phenomena, from ecosystems to social systems to the solar system. Examples of applications to disciplines farther afield than the traditional ones include psychology, history, and hermeneutics [27], [29], [31].

Despite the centrality of precision in the field of mathematics, the novelty and quick growth of chaos theory have left no universally accepted definition of "chaos." (It is somehow appropriate that the process of defining mathematical chaos has been somewhat chaotic.) The one feature appearing in anyone's definition is "sensitive dependence on initial conditions." Any uncertainty in the initial state of a given system can lead to rapidly growing errors in any effort to predict a future state of the system. That is, in a chaotic system, any point has another point arbitrarily close to it whose future is eventually drastically different from its own. Two neighboring water molecules have different fates when they're the ones going separate ways at a fork in a river. In a chaotic system, *every* point in the system exhibits that sort of eventual divergence from nearby points.

Although it was not called "chaos" for decades, Poincaré described the phenomenon in his essay *Science et méthode* in 1908:

> A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance. If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation *approximately*. If that enabled us to predict the succeeding situation *with the same approximation*, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon [36].

The implications of this phenomenon were not embraced by the mathematical and scientific community until much later in the twentieth century, stimulated by results of computer investigations.

Meteorologist Edward Lorenz rediscovered "sensitive dependence on initial conditions" in 1963 in a system of three linked nonlinear differential equations for modeling weather [17]. The term "The Butterfly Effect" has been widely used to describe the phenomenon, based on a paper presented by Lorenz in 1972 entitled "Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?" [18]. The motion of the air or water molecule is completely determined, from centuries-old Newtonian mechanics; yet, the slightest change in its starting position, from the flap of wings, can result in a very different itinerary. Since no physical situation can be known to infinite accuracy, behavior in chaotic systems eventually diverges from the predicted, not because the rules are wrong or even incomplete, but because our *knowledge* about the starting position is *necessarily* incomplete. It is not the case of probabilistic effects; chaos is *not* the same as randomness.

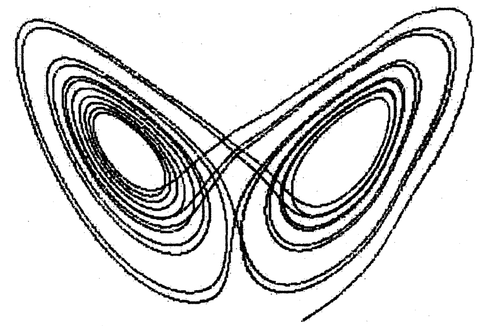

Figure 3: *The Lorenz attractor.*

The term "chaos" in this mathematical sense was first used in the paper "Period Three Implies Chaos" in a 1975 issue of the widely read *American Mathematical Monthly* [16]. The wide readership of the *Monthly*, the catchy term "chaos," and increased use of computers to investigate long-term behavior of dynamical systems combined to spread the newly rediscovered topic through the mathematical community.

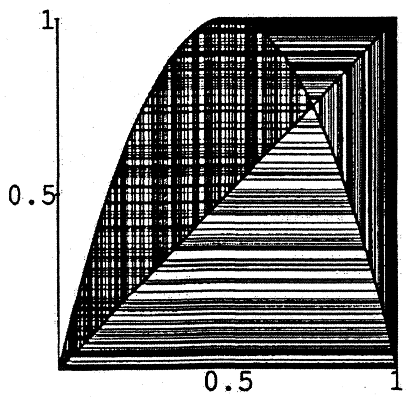

Figure 4: *Transitivity.*

Two other increasingly accepted requirements for a system to be called "chaotic" are topological transitivity and density of periodic points [8]. Briefly, a dynamical system is called topologically transitive if, given any two open subsets of the space, the image of one subset under the deterministic rules of the system will eventually intersect the other. That is, the system cannot be decomposed into dynamically distinct sets because one will always eventually intersect the other, for *any* pair of open subsets of the space. Periodic points are dense in a space if every point in the space has another point arbitrarily close to it that is periodic, which means it eventually comes back to itself exactly and repeats a pattern periodically. That is, no matter how "chaotic" or seemingly random and pattern-free a given point's orbit may be, *arbitrarily* nearby at *any* point along its trajectory lies a periodic point, one that repeats a finite itinerary indefinitely. Mathematical chaos, then, has three defining characteristics: unpredictability, indecomposability, and an element of regularity.

Is the universe chaotic, in some sense analogous to the mathematical definition? Do mathematicians see chaos in the local systems of their experience? Are our lives chaotic trajectories? Whether life is increasingly chaotic in the traditional sense or we are simply gaining a better understanding of the truly chaotic nature of the universe, one may find reassurance in these three aspects of chaos as metaphors for life's trajectory, both individually and collectively.

Individually, if one's own life is a path in a chaotic space, "sensitive dependence" suggests that change is always possible. Even if there are rules governing what comes next in the universe or to the individual, the application of those rules is very sensitive to changes. When one needs to make changes in one's path, the slightest variation can lead to very different outcomes – still not necessarily predictable because of our imprecise knowledge of the current state of the system. "Sensitive dependence" also suggests that each one of us has a unique path, despite similar initial conditions. Two life trajectories with very similar starting points (not just physical location but in the multidimensional space of existence) may end up with very different itineraries. It is determinism without the predestination, free will with a difference.

The element of regularity provides some balance to the consequences of sensitive dependence. Arbitrarily close to a chaotic trajectory is a periodic one; a seemingly wandering life trajectory can always have some hope of regularity within reach. ("Within reach," or arbitrarily close in our state space, does not guarantee that it is easily found.) Conversely, if life is periodic in the sense of going nowhere, if there *are* chaotic trajectories, one *may* be nearby. Regularity gives one hope of patterns always nearby, symmetry within one's chaos, understanding within one's reach. Topological transitivity suggests that there is always a trajectory that can take one from a neighborhood of a given "here" in one's life space to a neighborhood of some particular "there," in some finite time. This can give hope that perseverance may have its reward (although that finite time may be longer than one has at one's disposal).

Collectively, "sensitive dependence" suggests that same mutability for family or community or society or universe. Small actions can have large consequences. The acts of one person can actually make a difference. Change is possible, significant change. Regularity can give similar hope of patterns always nearby, hope for a reachable calm near what may seem an intractable storm for circles narrow or wide. The indecomposability from topological transitivity suggests that no one is dynamically isolated, that we are all connected somehow: certainly not uniformity, maybe not unity, but together somehow, in community.

More abstractly, there may be theological reassurance in this extended chaotic metaphor. It does not bridge the traditional gap between C.P. Snow's "Two Cultures" of the sciences and humanities or Stephen Jay Gould's "nonoverlapping magesteria" of science and religion [30], [13]. On the other hand, it does allow for the determinism of Descartes' Clock-Maker God, tempered by a mechanism of surprising complexity and unexpectedly plentiful possibilities. It is somewhere between random and rigid, mechanical and magical.

Some scientists have projected that historians will see the scientific ideas of the twentieth century as best represented by three aspects of uncertainty: Gödel's Incompleteness Theorem, Heisenberg's Uncertainty Principle, and chaos theory [19]. Gödel's Theorem states that for any mathematical system, there will always be propositions that can not be proven either true or false using the rules and axioms of that system. Briefly, for any axiom system, provability is a weaker notion than truth. (That sounds like a mathematical result that may lend itself to theological metaphor.) Heisenberg's "principle of indeterminacy" rules out a complete knowledge of the position and momentum of any subatomic particle: knowledge of one necessarily limits what can be known of the other. Gödel's Theorem places some theoretical results out of reach (but not most), and Heisenberg's subatomic particles seem far removed from daily existence. Chaos theory remains as uncertainty both ubiquitous yet tempered.

Mathematician Ralph Abraham sees chaos as "inescapable in the rhythm of the planets, the climate, the metapatterns of history." It is "the essential chaos of life, which is necessary for evolution and is the source of all creativity" [1]. Nineteenth century historian Henry Brooks Adams says, "Chaos often breeds life, when order breeds habit" [6]. In fact, two contemporary applications of chaos theory suggest just this necessity of chaos: a *lack* of chaos in either brainwave or heart rate readings can suggest pathology. Unbroken regularity can be interpreted as missing the creative spark.

Throughout cosmogony, from the *Theogony* of Hesiod to the Genesis story, the contrasted forces are Chaos and Cosmos. The universe replacing chaos is presented not as order from confusion but as harmonious existence from "a trackless waste, a formless void." In *The Birds* of Aristophanes, from Chaos and the darkness comes Eros, or love, which then is the force responsible for all creation. In Genesis, the breath of God stirs the primordial waters of chaos into giving birth to all that is and turns a dirt clod into a living soul. Different cultures have found some version of "chaos" as essential to the origins of the universe as we know it.

If our example of "chaos" has been viewed as essential from creation narratives to contemporary physiology, then how much more essential has mathematics in general been throughout the development of human culture? Consider the following opinion of Freeman Dyson: "Western science grew out of Christian theology. It is probably not an accident that modern science grew explosively in Christian Europe. … A thousand years of theological disputes nurtured the habit of analytical thinking that could be applied to the analysis of natural phenomena" [9]. This statement may seriously overlook vast traditions of science, natural history, and philosophy in non-Western cultures, and its causal implications may or may not be faulty, but it does highlight a long and honorable tradition of bridges between the analytical thinking associated with mathematics and a philosophical discourse on human existence, nature, and transcendence. Contemporary mathematics educators and mathematicians alike owe their students and society at large an acknowledgement of this tradition and a continued nurturing of "the habit of analytical thinking" applied across the spectrum of human ideas.

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