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Fractal Geometry And Self-Similarity In Architecture: An Overview Across The Centuries

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Abstract

Fractal geometry describes the irregular shapes and it can occur in many different places in both Mathematics and elsewhere in Nature. The aim of this paper is to present an overview which involves fractal geometry and the properties of self-similarity in architectural and design projects. We will refer of the building's characteristics in different cultures (e.g., Oriental and Western culture) and in different periods (e.g. in the Middle Ages until today).

1. Introduction

For many centuries architecture has followed the Euclidean geometry and Euclidean shapes (bricks, boards, so) it is no surprise that buildings have Euclidean aspects. The symmetry in the temples and in the buildings helped to realize the engineering calculus.

On the other hand, some architectural styles are informed by Nature, and much of Nature is manifestly fractal. So perhaps we should not be so surprised to find fractal architecture [16]. As we shall see, fractals appear in architecture for reasons other than mimicking patterns in Nature. Our fractal analysis in architecture has been divided in two parts:

- on a small scale analysis (e. g., to determine the single building shape);
- on a large scale analysis (e.g., to study the urban growth and the urban development) [3, 4, 10, 27].

In the small scale analysis we have observed:

- the box-counting dimension of a design, to determine its degree of complexity [6];
- the building's self-similarity (e.g., a building's component which repeats itself in different scales) [27, 28].

In this paper we shall present an overview of the self similarity in the buildings in different periods and different architectural styles.

2. The self-similarity

A fractal object is self – similar if it has undergone a transformation whereby the dimensions of the structure were all modified by the same scaling factor. The new shape may be smaller, larger, translated, and/or rotated, but its shape remains similar [9, 20, 24].

"Similar" means that the relative proportions of the shapes' sides and internal angles remain the same.

Figure 1 shows an example of self similarity applied in the von Koch's snowflake, a geometric fractal object [32]. Our sense, having evolved in nature's self-similar cascade, appreciates self-similarity in designed objects. The fractal shapes and the self-similarity are known to the artists and to the architects in different periods and in different cultures.

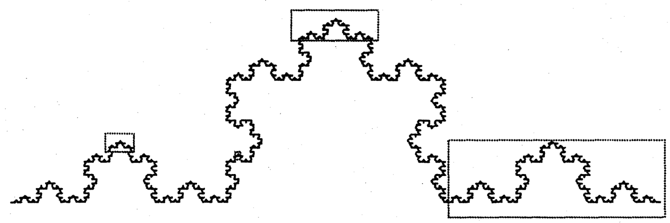


Figure 1: *The self-similarity in von Koch's snowflake.*

3. The self-similarity in architecture

We can classify the presence of the self-similarity in architecture using two different ways: unconscious, when the fractal quality has been unintentional chosen for an aesthetic sense, and conscious, when the fractal quality is in every case the result of a specific and conscious act of design. Conscious self-similarity appears in the modern architecture [11, 27, 28]. It is interesting to analyze the self similarity in different cultures and in different architectural styles.

3.1. Hindu architecture. For over two thousand years much of Asia has been dominated by Indian Hinduism as a religious, social and political force. Hindu Asia encompasses the subcontinent of India, the peripheral sub-Himalayan valleys, the major part of mainland South-East Asia and the Indonesian archipelago. The temple is the most characteristic artistic expression of Hinduism. The temple reflects the ideals and way of life of those who built it and whom it was intended to operate a link between the world of man and that of the gods. In order to understand the architectural forms of the Hindu temple it is necessary to investigate the origins and development of the civilization that produced it. In older cultures the mountains prefigure the sacred sanctuaries around the world. In the Hindu experience the idea of the archetypal mountain of existence is mythologized in the cosmic mountain named Meru, the mythological center or navel of the universe [22]. George Michell (1988) writes: «*In the superstructure of the Hindu temple, perhaps its most characteristic feature, the identification of the temple with the mountain is specific, and the superstructure itself is known as a 'mountain peak' or 'crest' (shikhara). The curved contours of some temple superstructures and their tiered arrangements owe much to a desire to suggest the visual effect of a mountain peak*» [21, p. 69].

The fractal structure of some mountains has been researched and discussed by analysts; self-similar angles of sloping stone are often observable once one has acquired "an eye for fractals". Indian and Southeast Asian temples and monuments exhibit a fractal structure. In fact, the towers are surrounded by smaller towers, surrounded by still smaller towers, and so on, for eight or more levels. In these cases the proliferation of towers represents various aspects of the Hindu pantheon [17]. The Hindu temple typically involves a multiple set of ideas. Perhaps Hindu traditional architecture has more symbolic meanings than the architecture of other cultures. It certainly is highly articulated. The temple is oriented to face East, the auspicious direction where the sun rises to dispel darkness. The temple design includes the archetypal image of a Cosmic Person spread out yogi-like, symmetrically filling the gridded space of the floor plan, his navel in the center, and it includes the archetype of the cosmic mountain, and the cave of sacred inner mystery, and other imagery as well (of the link between earth and heaven, of fertility, planets, city of the gods, deities, etc.). Quoting William Jackson: «*The ideal form gracefully artificed suggests the infinite rising levels of existence and consciousness, expanding sizes rising toward transcendence above, and at the same time housing the sacred deep within*» [17]. Figure 2 represents a fractal Indian temple.

Figure 3 shows an example of Moghul art: the "Taj Mahal" (Agra, India) (1632–1648). Its name is the deformation of "Muntaz Mahal" which means "palace's diadem". In this funeral mosque the self-similarity is present in the arches' shape repeated in four different scales (see figure 2). Humayun's Mausoleum at Dehli (India) (moghul art, 1557–1565) presents a descending fractal structure [12].

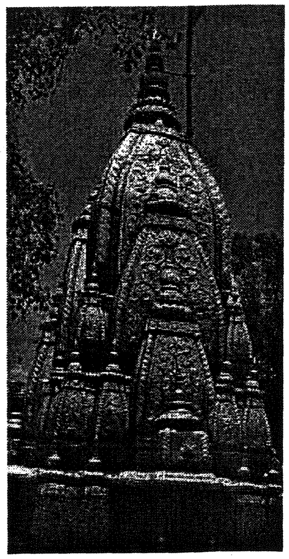


Figure 2: *Kasi Viswanath Temple, (Varanas, India).*

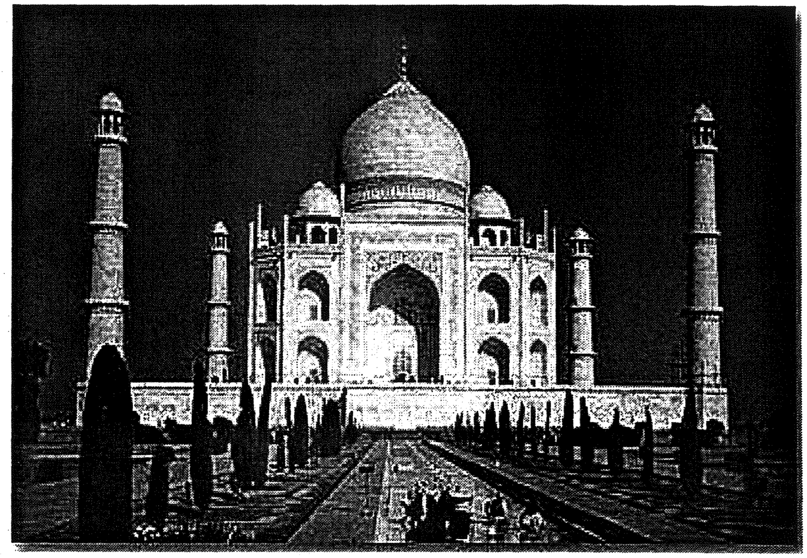
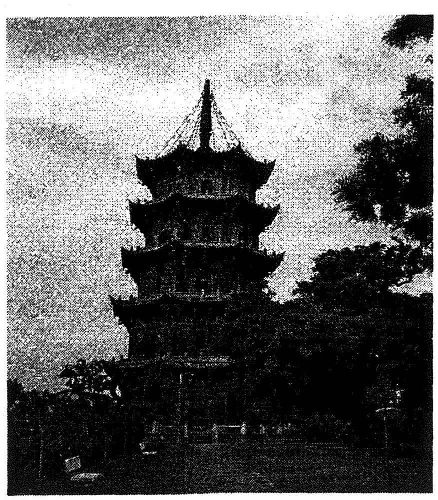


Figure 3: *The Taj Mahal (Agra, India).*

3.2. Oriental architecture. There are other examples of self-similarity in Oriental architecture [26]. Figure 4a illustrates the "Kaiyuan Si Pagoda", Chinese architecture (Song Dynasty, 1228 – 1250, Quanzhou, Fuqian). Observing Figure 4b, which represents Kaiyuan Si Pagoda's plan, we can note the self similarity in the octagonal shape [19]. Octagonal shape appears in other pagodas.



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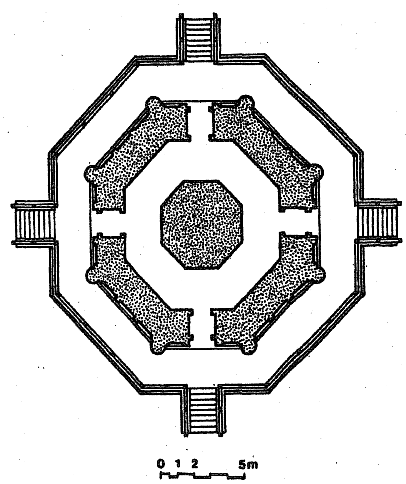


b)

Figure 4: *Kaiyuan Si Pagoda we can note the self-similarity in the shape (a) and in the plan (b).*

We can also find the presence of fractal geometry and the self-similarity in the "Sacred Stupa" Pha That Luang - Vientane (Laos), where the basic shape is repeated in different scales (see figure 5) [27], and in the Royal Palace of Mandalay (Burma) (figure 6).

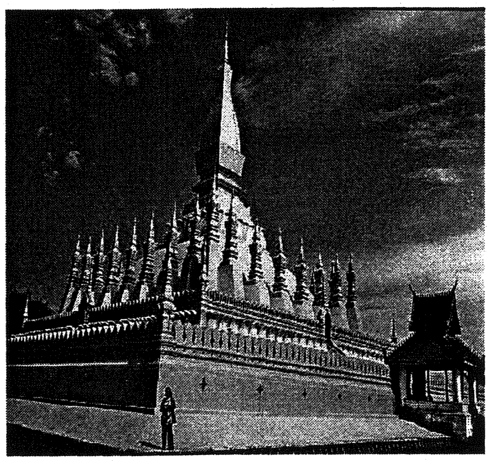


Figure 5: *The Sacred Stupa (Vientiane, Laos).*

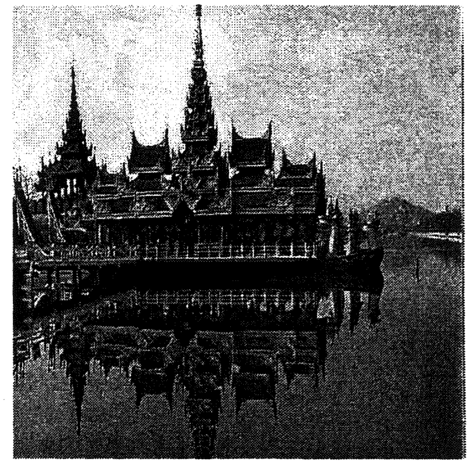
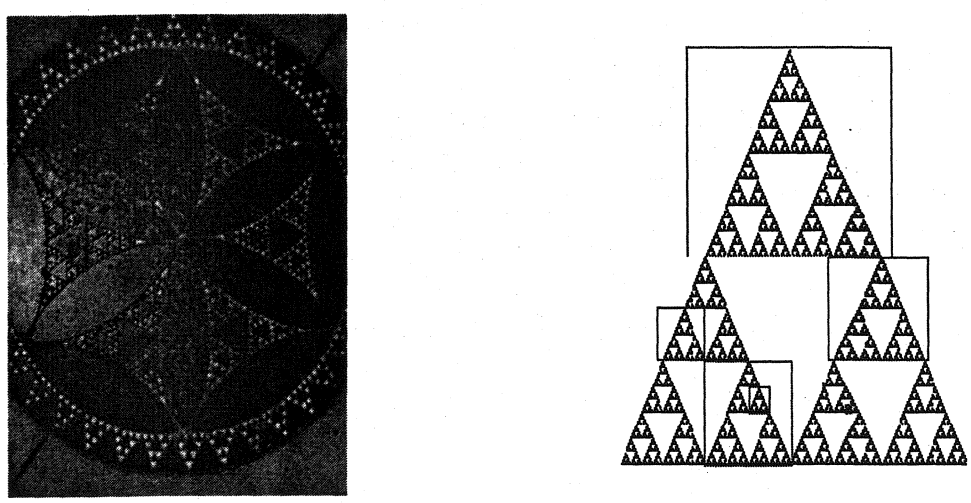


Figure 6 *Royal Palace (Burma) an example of fractal architecture.*

3.3. Western architecture. In the Western architecture we can find the oldest handmade fractal object in the Cathedral of Anagni (Italy) [27]. Inside the cathedral, built in the year 1104, there is a floor, illustrated in Figure 7a, which is adorned with dozens of mosaics, each in the form of a Sierpinski gasket fractal (shown in the Figure 7b).

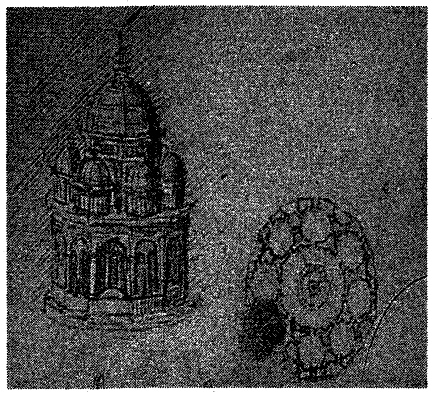


a) b)

Figure 7: *The floor of the Cathedral of Anagni a) and the Sierpinski gasket. b).*

The intricate decoration of Renaissance and Baroque architecture, especially as expressed in cathedrals, frequently exhibited scaling over several levels. The art historian George Hersey points out fractal characteristics in Bramante's 1506 plan for the new St. Peter's. This plan may be called fractal: it repeats like units at different scales [16].

Figure 8 illustrates the da Vinci (1452–1519) plan and the wooden model for a domed cathedral with nine cupolas. The self-similarity is present in the plan (the circles in four different scales) and in the building (two reduction scales in the cupolas) [28].



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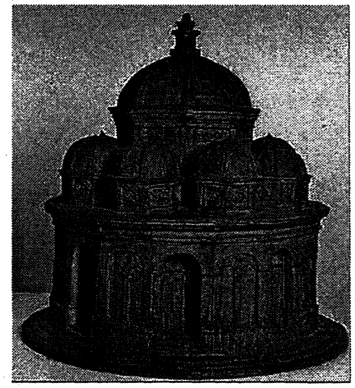
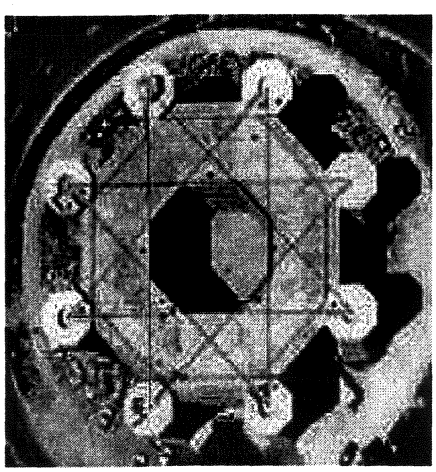


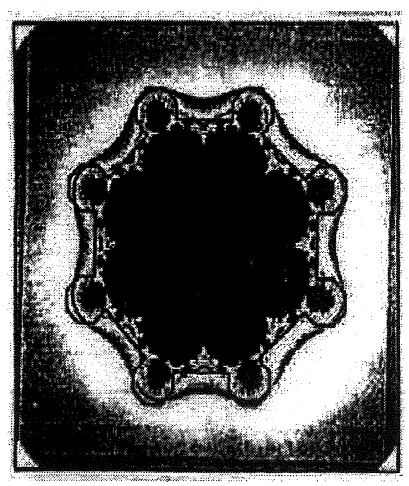
b)

Figure 8: *Da Vinci's plan and the model on wood for a self-similar cathedral.*

The fractal geometry is present in the "Castel del Monte" (Andria, Apulia, Southern Italy) built by the Holy Roman Emperor Friederich II of Hohenstaufen (1194 - 1250) in the last decade of his life. We can find a self-similar octagon in the plan (in analogy with the figure 4a). In fact, the outer shape is an octagon, as is the inner courtyard. Even the eight small towers have octagonal symmetry. It is interesting to note that it is possible to find a connection between the Castel del Monte's shape and the Mandelbrot set (figure 9b). Castel del Monte has other interesting implications with geometry, in fact the planimetric aerial photo (see figure 9a) shows that the tangents of the octagon forming the inner courtyard intersect at the center of the octagonal corner towers. This involves a geometric relationship between the towers and the inner courtyard, established by the similarity describe in a Gotze's work [13, 14]. There is also the presence of the golden ratio [23, 27, 29].



a)



b)

Figure 9: *The connection between Castel del Monte (a) and a Mandelbrot set (b).*

3.4. Modern architecture. The conscious building's self-similarity is a recent discovery by the twentieth century architects as a result of a specific and conscious act of design. For example, Frank Lloyd Wright (1867-1959), in his late work ("Palmer house" in Ann Arbor, Michigan, 1950-1951) has used some self-similar equilateral triangles in the plan. A kind of "nesting" of fractal forms can be observed at two point in the Palmer house: the entry way and the fireplace. At these places one encounters not only actual triangles but also implied (truncated) triangles. At the entrance there are not only the triangles composing the ceramic ornament, there is also triangular light fixture atop of triangular pier. There is a triangle jutting forward overhead. The fireplace hearth is a triangular cavity enclosed between triangular piers. The concrete slab in which the grate rests is a triangle. The hassocks are truncated triangles [11].

Remembering the definition of the fractal as « *a geometrical figure in which an identical motif repeats itself on an ever diminishing scale*», the Palmer house is an excellent illustration of this concept. Other Wright's example of fractal architecture are the "Robie house" and the "Marin County Civic Center", San Rafael (1957) where the self similarity is present in the external arches. Figure 10 shows Wright's Marin County Civic Center and figure 11 illustrates a Roman aqueduct, the analogy in the shape is amazing.

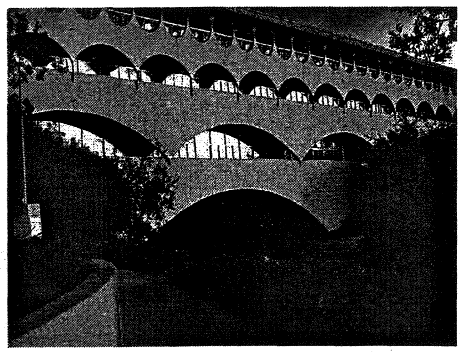


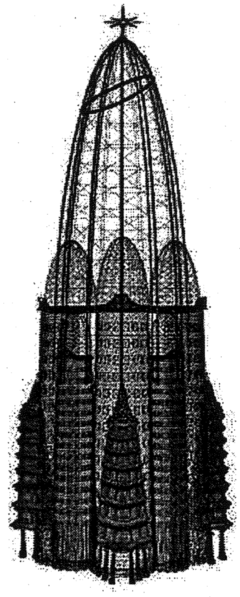
Figure 10: *Wright's Marin County Civic Center (San Rafael, USA)*

Figure 11: *Roman aqueduct (20 B.C) (Nimes, France)*

Few people know that in 1908 the Catalan architect Antoni Gaudí (1852-1926) imagined a skyscraper for New York City. The building was drawn sometime between 1908 and 1911 having been ordered by an unknown American businessmen wanting a big hotel for New York. However, the project was never realised; it got lost within the time and fell into oblivion. Only some original sketches survived as well as some drawings by sculptor Llorenç Matamala i Pinyol, friend and collaborator of Gaudí. The building, that Gaudí traced in 5 minuscule sketches on card paper, was an enormous construction that would have been the biggest of New York City at the time: 360 meters in height, something less than the Empire State, built in the 1931,and 60 meters less than the remembered Twin Towers. The shape of this rugged tower, in figure 12, would have been reminiscent of the temple of the "Sagrada Familia" (located in Barcelona, Spain), and it is similar to that of the Hindu temple shows in figure 13.

Kazimir Malevich (1878 – 1935) was an important figure in Russian and Soviet art and architecture in the early 20th century. Largely self-educated, Malevich was from the beginning of his artistic career, «*not concerned with nature or analyzing visual impressions, but with man and his relation to the cosmos*» [15, p. 145]. Much of his work belongs to the Suprematist school. During the 20s, he began expressing architectural projects as 3-dimensional sculptures. Some examples of this Arkhitektonics are marvellous instances of fractals in architecture.

Malevich creates buildings with ambiguous scales, erasing the difference in scale between buildings and people. This is achieved by surrounding the largest component of a building with a cascade of smaller and smaller copies, number and scale governed by an approximate 1/f relation (see figure 14).



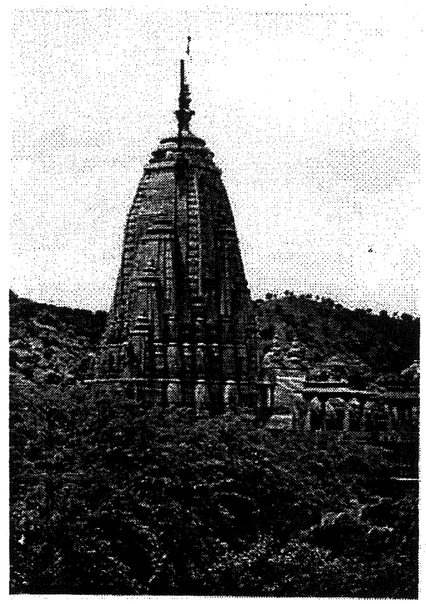


Figure 12: *Gaudí's skyscraper (1908).* **Figure 13**: *Hindu Temple.*

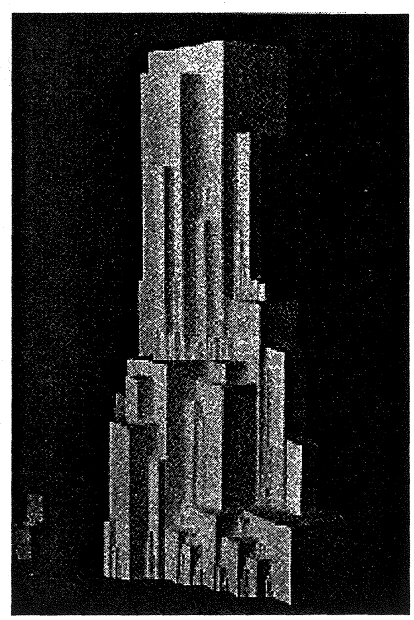


Figure 14: *A Malevich fractal building.*

Italian architect Paolo Portoghesi has used the conscious self-similarity to realize the "Chamber of Deputies" (Rome, Italy, 1967), shown in figure 15, where we can observe some self-similar spirals, and the "Villa Papadanice" (1966) with the presence of self-similar circles in the plan (figure 16) [25].
Spanish architect and engineer Santiago Calatrava has realized futurist buildings and bridges. The complexity of his product engineering suggests a fractal interpretation of his projects. Figure 17 shows the

"Hemispheric", an amazing glass structure, realized in the "Ciudad de las Artes y las Ciencias" (Valencia, Spain), which evidences an interesting analogy with the shape of fractal cuirass of the armadillo, as shown in figures 18 and 19.

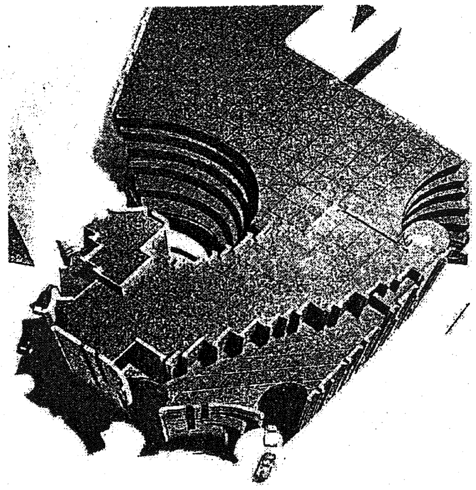


Figure 15: *Portoghesi's Chamber of deputies (Rome, Italy).*

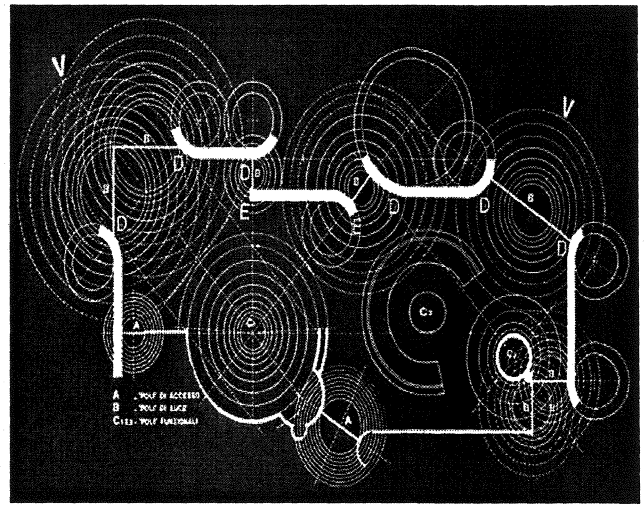


Figure 16: *Portoghesi's Villa Papadanice (1966) (Rome, Italy).*

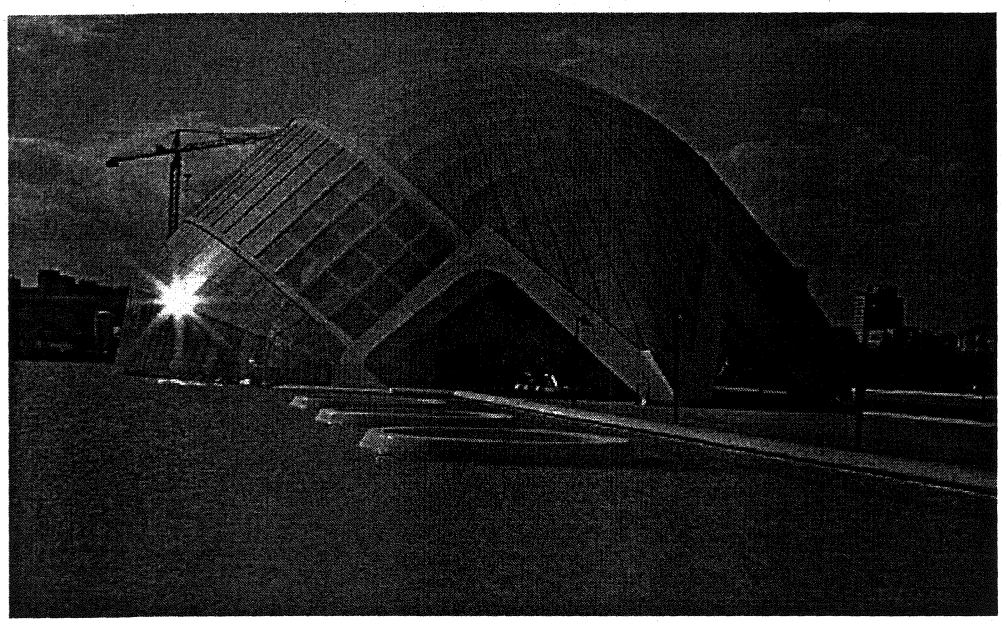


Figure 17: *Hemispheric (Valencia, Spain) by Santiago Calatrava.*

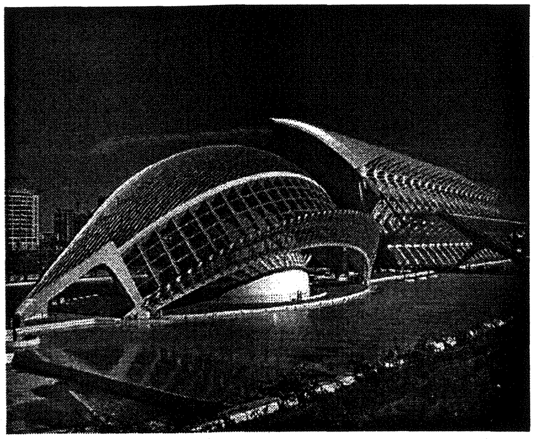


Figure 18: *Hemispheric (Valencia) by Santiago Calatrava.*

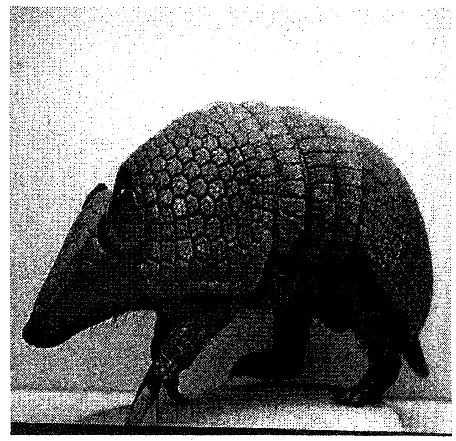


Figure 19: *An armadillo.*

4. Conclusions

Fractal geometry and its connection between the complexity and the Chaos theory can help to introduce the new complexity paradigm in architecture [5, 7, 8, 9, 10, 18]. Fractal distributions can be used to generate complex rhythms for use in design. As an example, the fractal dimension of a mountain ridge behind an architectural project could be measured and used to guide the fractal rhythms of the project design [4]. The project design and the site background would then have similar rhythmic characteristics. In both criticism and design, fractal geometry provides a quantifiable calibration tool for the mixture of order and surprise.

This paper has presented only some particular aspect of the self-similarity in the buildings, but we can also apply the fractal geometry in the urban growth [3, 4, 5]. In fact, the most exciting scientific developments of the past decade, such as fractals, complexity theory, evolutionary biology, and artificial intelligence give us an idea of how human beings interact with their environment. Organisms, computer programs, buildings, neighbourhoods, and cities share the same general rules governing a complex hierarchical system. All matter, biological as well as inanimate, organizes itself into coherent structures. The human mind has evolved in order to adapt to complex patterns in the natural world, so the patterns we perceive around us influence our internal function as human beings [30]. A new, human-oriented architecture follows ideas by Christopher Alexander, combining the best qualities of traditional architecture with the latest technological and scientific advances [1, 2, 31]. The skeletal structure of the industrial city is tree-like with radial street systems converging on the historic core. When these tree-like structure are embedded into urban development we begin to see typical patterns emerge which are clearly fractal [3, 4, 5].

Micheal Batty and Paul Longley, authors of *Fractal Cities* (1994), have introduced a fractal generation of the cities using the cellular automata models, and they have interpreted the results in a context of self-organization [5]. *Fractal Cities* is a pioneering study of the development and use of fractal geometry for understanding and planning the physical form of cities, showing how this geometry enables cities to be simulated through computer graphics [4]. The greatest architecture is complex and coherent; but neither random, nor simplistic. By understanding how to generate "life" in built structures, we can improve the way buildings and cities relate to people. Unfortunately, the universe's wonderfully rich complexity is ignored and suppressed by a contemporary design canon that seeks plainness and a false purity.

The fractal geometry and the self-similarity are helping to define a new architectural models and an aesthetic that has always lain beneath the changing artistic ideas of different periods, schools and cultures.

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