BRIDGES

Mathematical Connections in Art, Music, and Science

Leonardo da Vinci's Bar Grids

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Abstract

In 1989, I began constructing domes using notched bars assembled according to a simple rule. This led me to explore planar constructions based on this rule using fixed length "notched" line segments. I was able to create a wide variety of patterns. Based on certain of these patterns I was able to construct spheres and cylinders from notched curved rods without the use of glue, rope, nails, or screws. Surprisingly, drawings found in the notebooks of Leonardo da Vinci suggest that he too explored constructions based on this simple rule.

1. Bar Grids

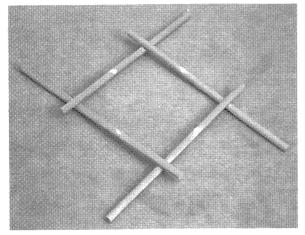
In 1989 I found a way to build domes using simple rods in such a manner that the rods require no fastening materials like nails, screws, wire and so on.

The construction admits a simple description. So we start with a number of rods. On each rod we determine four points as indicated in figure 1. We call these points connecting points. We distinguish two types of connecting points: End points (closest to the ends of the rods and interior points (the remaining points). So each rod has two end points and two interior points.



Figure 1: Position of the four connecting points.

In constructing the dome we now apply the following rules: one of the endpoints of a rod is placed on a free interior point of a different rod. At the end all connecting points of the rods have to be used as a connection between two rods, except near the border of the construction. Now the actual construction of the dome turns out to be a simple task. Beginning with four rods as in figure 2 we extend the construction by continually adding rods at the bottom (see figure 3). Since we add one rod at the time, on the outer edge, the dome can be constructed by one person. The four poles with which we have started will rise automatically during the building process and at the end the dome, consisting of 64 rods rests on the earth with only 16 rods (see figure 4).



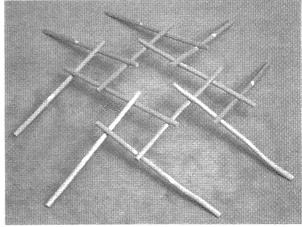
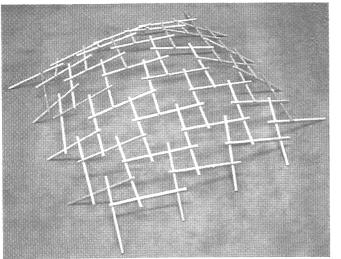


Figure 2: Dome construction (a).

Figure 3: Dome construction (b).



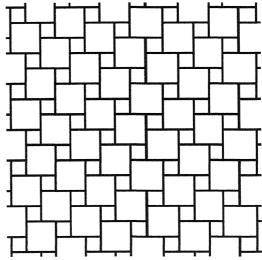


Figure 4: Dome construction (c).

Figure 5: Bar grid.

With the above construction process various patterns can be formed, each leading to a domelike construction. In the sequel we will call the patterns that can be formed with the above rules bar grids. The bar grid of the dome of figure 4 can be drawn simplified as in figure 5. In this form the drawing looks like a tiling pattern. However, we are not interested in the tiles but in the joints between the tiles. So we have a grid consisting of straight lines representing the rods. A first investigation into the various possible bar grids soon resulted into dozens of patterns, some of which are shown in figure 6.

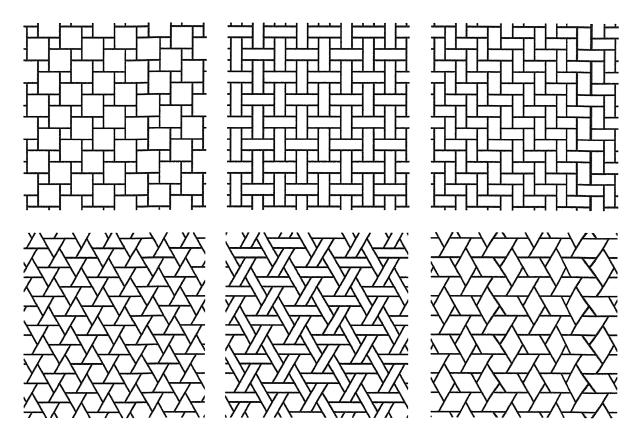


Figure 6: Examples of bar grids.

2. Leonardo da Vinci

Since the system is so simple, I could not imagine that it had not been invented before. The first name that came into my mind was Buckminster Fuller. In his work I found related drawings of patterns, however the constructions derived from them are all made from rods and wires (tensegrity). In the end I have found only one comparable source. On a page from one of the notebooks by Leonardo da Vinci, Codex Atlanticus f. 328 v-a, we find among others three patterns with exactly the properties of the bar grids defined above (see figures 7, 8, 9).

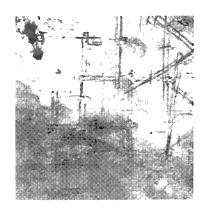


Figure 7: Pattern 1.

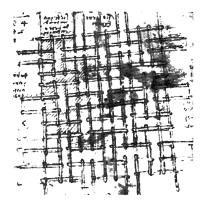


Figure 8: Pattern 2.

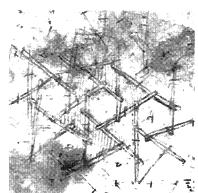


Figure 9: Pattern 3.

A reprint of this page can be found in Carlo Pedretti's book Leonardo Architect. As a description of the contents of this page Carlo Pedretti gives: "Studies of wooden roofing made up of parts that fit together." And in the text it is described as: "Geodesic' roofing for vast area of land, anticipating the daring constructions of Buckminster Fuller". In view of the way in which the patterns are drawn, oblong forms that seem to lie on each other, the most direct interpretation is that here we have to do with stacking construction build from straight rods. On making a model this leads exactly to the domes that I found. So the conjecture seems justified that Leonardo da Vinci is the first inventor of these constructions, although we cannot be sure about this.

3. Spheres

In the domes it is gravity that keeps the loose rods together. It follows that continuing the construction as far as a complete sphere is not possible. Yet it turns out that using the above construction system objects can be formed where only the elements themselves, instead of gravity, keep the construction together. For example, we can assemble a sphere from a number of rods, or more generally elements, without using connecting materials like wire or glue. The number of connecting points per elements and the connecting rules do not change. It is only the form of the elements that changes. For a sphere we use curved rods instead of the straight rods for a dome.

A simple way to come to a design for such a sphere shaped construction is the following: in the bar grid of figure 10 the midpoints of the hexagons are connected such that a pattern of triangles results (figure 11). Eight of these triangles can be used to form an octahedron (figure 12).

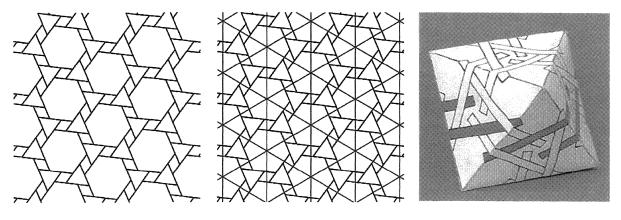
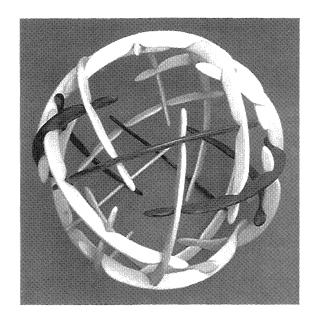


Figure 10: Bar grid

Figure 11

Figure 12: Octahedron.

On this octahedron we now see a grid consisting of 24 bars and this can be used a design for the sphere of figure 13. The form of the elements has been determined such that no tension arises in the sphere. Only when closing the sphere non-rigidity of the elements is required. The relative position of the elements causes the sphere to stay in one piece: each of the elements is prevented to fall by other elements. For the sphere of figure 14, which consists of 90 elements, the icosahedron has been used as an intermediate step so that pentagons occur in the construction.



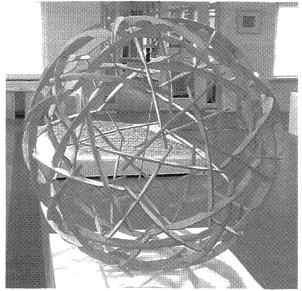
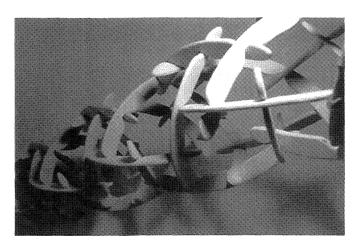


Figure 13: Sphere - 24 elements.

Figure 14: Sphere - 90 elements.

4. Other Shapes

Beside domes and spheres other shapes have been realized like cylinders (figures 15, 16).



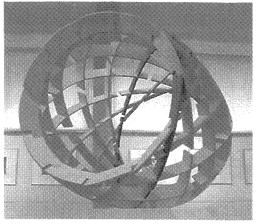
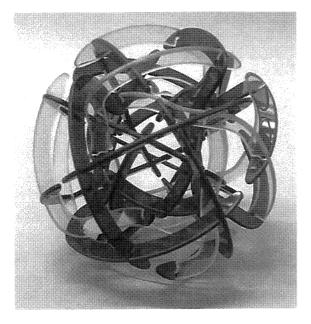


Figure 15: Cylinder (a).

Figure 16: Cylinder (b).

A real new step was made at the development of objects in which the inner space of the sphere is used too, as in figure 17. This object has the form of two linked concentric spheres. The whole is a stable construction consisting of 24 elements. Each element is halfway (that is to say with two out of four connecting points) in the outer sphere and halfway in the inner sphere. The object of figure 18 is a different example; it is composed of 12 elements. Three of the elements leave the outer surface of the sphere with its middle part leading to an intriguing structure.



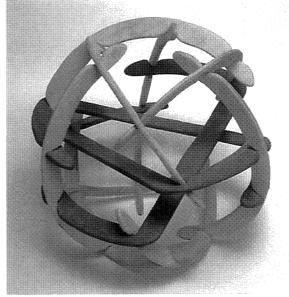


Figure 17: Concentric Spheres.

Figure 18: Inside the Sphere

In June 2003, there will be a seminar held at the da Vinci Museum at Vinci, Italy to explore the relationship between Leonardo's sketches and the constructions shown. We will attempt some large-scale constructions using notched poles at that time (see figure 19).

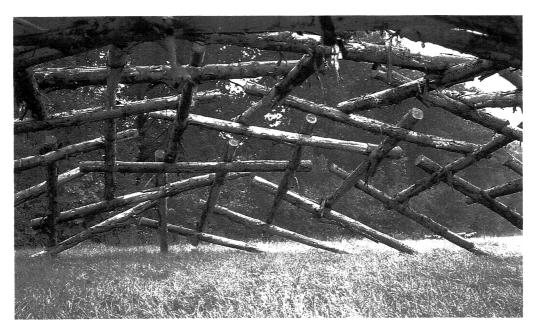


Figure 19: Dome with poles.

References

[1] Carlo Pedretti, Leonardo Architect, pp. 154-155. 1981.