BRIDGES
Mathematical Connections
in Art, Music, and Science

Mathematics for Those Who Hate Mathematics

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Abstract

Students in liberal arts mathematics classes seem to have one defining characteristic – a lack of confidence in their mathematical ability. Their main goal on the first day of class is to simply make it to the last day of class with a grade that will sufficiently show that they are mathematically competent, so that they will never have to take another mathematics course for as long as they live. Having to teach two or more classes of 35 to 40 students each with this type of attitude is a challenge. One solution I have found is to teach the entire course with a base in the arts. I have used this approach in teaching classes to students from age 7 through college, and the results are almost unanimous. While the students may not admit that they "like" mathematics at the end of the course, most will admit to appreciating it. And most will also admit that just hearing the word "mathematics" doesn't strike fear in them the way it did on the first day of class. To me, this is a success. This is why I teach. To catch that glimmer of appreciation in an otherwise indignantly closed eye. This paper will share some of the topics and activities that I have used in my classes, as well as some of the outstanding results.

Foundations

I had hoped that the students would be open to the hands-on, activity-based approach I take in my liberal arts mathematics courses, but I was in no way prepared for the extent to which they embraced it. The prerequisites for my course are nil. It is assumed that they have placed above the 9th grade algebra skill level on our university mathematics placement exam. It is the content and my students' views and work in this course that I would like to share in this paper.

I begin the course on the first day by asking how many students dislike mathematics. Hands rise from all corners of the room. The students begin to realize that they are not alone in their feelings on the subject, and this is extremely important at this level. A discussion of why they feel this way usually leads to their admission of a disillusionment as to why they have had to learn concepts, theorems, and formulas, and practice calculations over and over and over again when they will never need them in real life. Another strong factor behind their disinterest and lack of confidence stems from specific poor past experiences: a concept never mastered, a teacher with a harsh word, a parent with the inability to understand or offer assistance. All of these experiences lead them to the present. They don't like mathematics and they don't want to be sitting in my classroom. The challenge begins.

First, I promise them that they will not be required to study any algebra or geometry as they have in the past, i.e. no factoring and solving equations, no theorems and proofs. Next, I tell them that the only requirement for them to stay in the course is that they have an open mind. The course is very activity-based, interactive, and hands-on. Students are surprised to learn that mathematical operations do not only include addition, subtraction, multiplication, division, and exponentiation, but also movements such as translations, rotations, and reflections. I explain to them that they will learn to understand how these operations are applied through manipulation and discovery. Finally, I approach the grading policy. There are no exams. This usually makes them happy! Students' knowledge of the subject matter is assessed through their ability to take what they have learned, apply it to specific examples of their choice, and explain the results in writing. Three papers, called Synthesis Projects, were assigned this past semester (Spring 2002), each focusing on a different category within "the arts" to allow students to apply their knowledge in a wide variety of areas. Synthesis project topics included studying the mathematical influence in another culture, in architecture, and in visual art, literature or music. Smaller projects, called Discovery Projects, were assigned to test knowledge of one particular topic, such as the Fibonacci sequence or the Golden Rectangle. Many of these Discovery Projects also required written summaries, although some were simply activities. Two Field Projects allowed students to apply their knowledge to activities outside of the classroom. They were required to attend one art, music, or theatre event and one sporting event and explain how mathematics played a role in some aspect of the performance. The culmination of the semester was a final project and poster session for which the students chose their own topics to analyze mathematically and present to the class.

This past semester, the course began with the students listing all of the categories that they felt fell into "the arts". Those categories included music, drama, literature, sculpting, photography, graphic design, television, poetry, theatre, pottery, sports, food, architecture, painting, filmmaking, language, performance, composition, interior design, teaching, dance, karate, drawing, textile design, fashion design, printmaking, animation, lyrics, body art, and landscaping. Students were encouraged to think about any or all of these and similar topics as the discussion continued during the semester. Topics covered in the class generally include Recursive rules and the Fibonacci and Lucas sequences; the Great Pyramid and Squaring the Circle; the Golden ratio, rectangle, and spiral; Symmetry types; Rosettes; Strip patterns; Wallpaper patterns; Surrealism and Point transformations; Optical Art and Illusions; Regular tessellations; Perspective; and Fractals. A discussion of how I approach some of these topics follows.

Course Content

Fibonacci Sequence and the Golden Ratio. Students in this course require a lot of encouragement and need a boost in their mathematical confidence. One way this is achieved is by allowing them to "experience" mathematics, rather than forcing them to perform calculation after calculation. This does not mean that all of the mathematics presented to them needs to be on an extremely elementary level. An example of this was our discussion of the derivation of the golden ratio, ϕ, from the Fibonacci sequence. Students were first presented with the idea of a recursive rule used to generate the Fibonacci sequence and were asked to list the first 20 terms of the sequence. Having them stop the recursive rule when 20 terms were determined allowed us to discuss the difference between a finite sequence and an infinite sequence, as well as what particular differences would be required in the steps of a recursive rule for each type of sequence. Next, the notation F_n was discussed and students were asked to identify the terms in their sequence using the notation. Then we discussed the meaning of F_{n+1} relative to some F_n. Finally, students were asked to calculate the ratio F_{n+1}/F_n accurate to 4 decimal places.

$F_2/F_1=1/1=1$	$F_{12}/F_{11}=144/89=1.6180$	$F_{18}/F_{17}=2584/1597=1.6180$
$F_3/F_2=2/1=2$	$F_{13}/F_{12}=233/144=1.6181$	$F_{19}/F_{18}=4181/2584=1.6180$
$F_4/F_3=3/2=1.5$	$F_{14}/F_{13}=377/233=1.6180$	$F_{20}/F_{19}=6765/4181=1.6180$
$\vdots$	$\vdots$	$\vdots$

At four decimal places of accuracy, the values of F_{14}/F_{13} and F_{20}/F_{19} appear equal. A discussion ensued about whether this is actually the case or not, along with a discussion of the relative change in the ratio as n is increased from 13 to 14 versus 19 to 20. This led to a conjecture on the students' part that the ratio may be an irrational number. We also discussed the idea of the ratio as a limit. Seeing the entire progression of ratios developed from the sequence, the students could easily see that the sequence was infinite and readily offered a solution to F_{21}/F_{20}. Because they calculated the ratios themselves, and observed the trend as it occurred, they were more likely to understand the concept of a limit. I finished this exercise by explaining to them what they had actually done, which was to answer a question that appeared to be out of a Calculus II class:

$$\lim_{n \to +\infty} \frac{F_{n+1}}{F_n} = \phi .$$

Golden Rectangles and Spirals. The students' confidence began to grow. They were eager and willing to learn the mathematics, they just didn't want it to appear that they were learning mathematics. I have learned that in order to keep my students at ease, I must use a subtle approach when introducing new mathematical concepts into this class. A discussion of the golden ratio led to the golden rectangle. We viewed slides of Leonardo da Vinci's *Mona Lisa* and the Parthenon and we discussed how golden rectangles are prevalent in each, and whether this was intended or not. Then we slowly approached the question of their construction. By now, the students understood the characteristics of an irrational number, primarily that the decimal part neither repeated nor terminated. But then I asked them to construct a "true" golden rectangle. Could they simply draw one using four lines while maintaining the correct length-to-width ratio? Some referred back to the Fibonacci sequence and suggested constructing a 5x8 rectangle or a 55x89 rectangle. This led to the development of a definition of an "approximate" golden rectangle versus a "true" golden rectangle, and at what point is the estimate "good enough"? They soon realized that the problem lay in measuring a length that was irrational. Using a square as a base, the students learned to construct a rectangle that is golden without the use of numerical measurements.

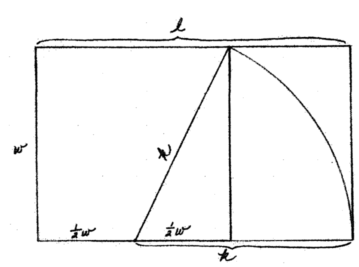


Figure 1. Golden Rectangle constructed from a square. $\frac{l}{w} = \phi$.

This is a course whose components seem to the students to be disjoint at times, with the only common factor being the application to the arts. While students may dislike the nature of a cumulative final in an algebra class, sometimes they also tend to dislike the segmented nature of this course. Accordingly, I continually try to bring topics from previous weeks back into the present discussion. The exploration of examples of the golden rectangle led to the nautilus shell and to the construction of a logarithmic spiral, which was done as an in-class project. After the students had mastered the construction, I asked them to look at it again, and write a recursive rule that would generate the spiral. This allowed them to unify two ideas into one small project.

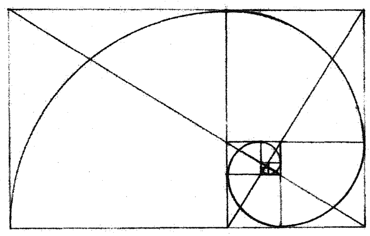


Figure 2. Golden Spiral.

Rosette Patterns. One of the greatest fears that student have upon entering my class, aside from their mathematical ability, is their artistic ability. I do require that they create designs and some assignments do require that they draw. Giving the students a wide berth when it comes to artistic ability allows them to relax. Students are encouraged to try to move beyond the "stick figure stage" as I refer to it in class, but they are also assured that I don't expect masterpieces either, and that I do not grade on artistic ability, but rather on their ability to incorporate the concepts into the assignment. For the most part, they create simple designs. However their results were better than I could have imagined. In the study of rosette patterns, I asked them to design two rosettes: one dihedral rosette to be classified as D_6 (showing 6 lines of bilateral symmetry and having 60° rotational symmetry) and one cyclic rosette to be classified as C_8 (showing no lines of bilateral symmetry and having 45° rotational symmetry). And while some of the designs were very simple, such as a plus sign + centered on each of 6 equal wedges of a circle for the D_6, many of the designs, such as those shown below, were much more intricate, and took much more time, preparation, and thought.

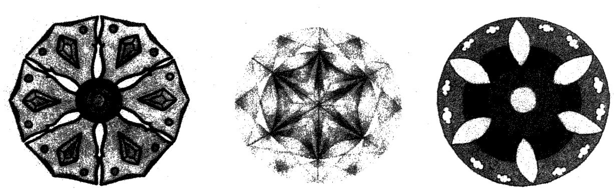


Figure 3. Three dihedral rosettes each showing 6 lines of bilateral symmetry and 60° rotational symmetry.

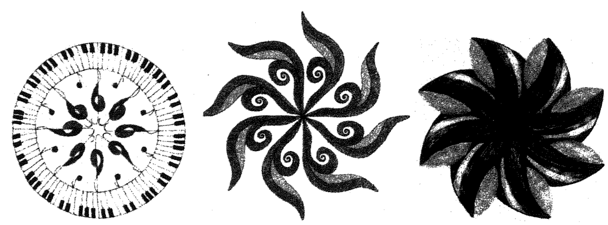


Figure 4. Three cyclic rosettes each showing no lines of bilateral symmetry and 45° rotation.

One-dimensional Patterns. We also explored one-dimensional border patterns and looked at several examples. We classified them by their International Crystallographic Union code, using the flowchart for strip patterns found in **[1]**. Classification is based on existence of vertical and horizontal reflection lines, half-turn symmetries, and glide reflection. Although the combinations of these symmetry types seem endless, students learned that there are only seven distinct classifications of border patterns possible.

Once familiar with the seven different types of patterns, students were asked to design their own border pattern. Once the design was complete, they were to use the flowchart to classify the pattern and have another student verify their findings. The artistic results were wonderful, and the most surprising part was the students' reactions when they discovered that many of their designs were classified as having no vertical or horizontal reflection, no glide reflection, and no half-turn. They were actually upset to discover that they designed something with "nothing interesting in it", as they put it! Some examples of student work on border patterns can be seen below.

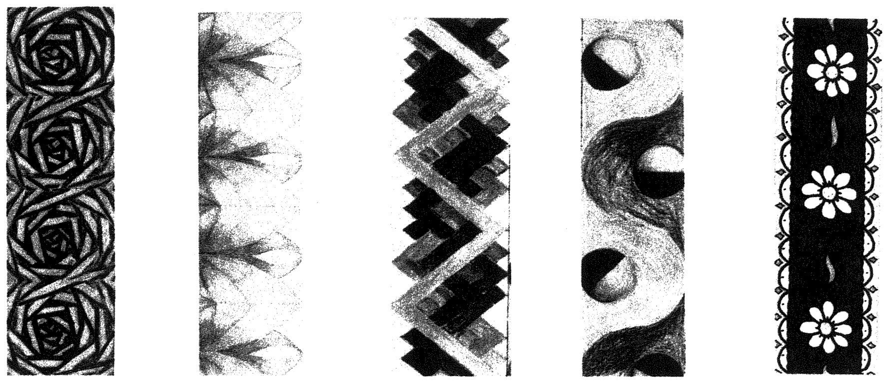


Figure 5. Student Border Designs

Two-dimensional Patterns. This past semester, I had the opportunity to teach two of the best groups of students I have ever had in this particular course. They were extremely receptive to the ideas presented and very appreciative of the hands-on approach, despite the fact that they felt they had little or no artistic ability. Their enthusiasm towards the border design unit led them to choose to work through the two-dimensional wallpaper design unit as well. I had never had a class opt to attempt this unit. Again, once familiar with the two-dimensional classification flowchart found in **[1]**, which classifies these types of patterns in a manner similar to that of the border design flowchart, the assignment was for the students to create their own wallpaper designs.

I always encouraged the students to create designs that they liked, rather than designs that would have some particular mathematical property. But what I noticed during the activity for this unit was that the students, having previously constructed their self-described "non-interesting" border designs, were actually drawing wallpaper designs, coding them using the flowchart, and then if necessary, modifying the designs to have "some" type of symmetry, and thus allowing the final design to be classified as something "better than" p1, the classification reserved for patterns with no symmetry types present. The results, again, were terrific.

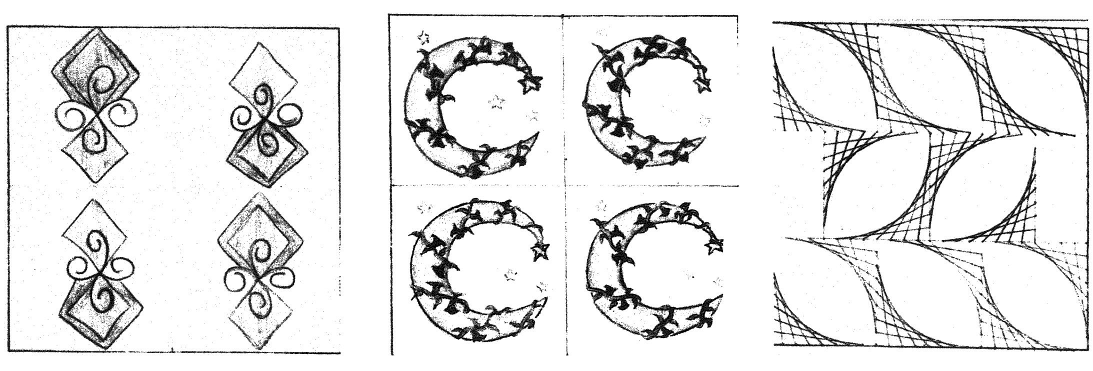


Figure 6. Student wallpaper designs

Tessellations. As part of the introduction of symmetry types, I showed a series of slides illustrating some of the tessellation works of M.C. Escher. Following the units on patterns, we approached tessellation construction. The unit began with a quick review of the basic regular polygons and a query as to which of the shapes the students thought would be able to tile the plane in a regular tiling. Students were asked to think about how angles of the polygons must interact with each other in order to successfully tile the plane. They discovered that it is the relationship between the interior angles of shapes that determines a successful tiling. As mine is a course that de-emphasizes the use of formulas to determine solutions, students were taught how to determine the measure of the exterior angles of a regular polygon, given that they are equal and their sum is 360°, in order to find the measure of the interior angles.

Once the students understood the types of polygons that had the ability to tile the plane, I had them create some tessellations of their own design. We discussed two different methods for creating their tiling templates: tiling by translation using either a parallelogram or a hexagon shape and tiling by half-turn using a triangle.

Tiling by translation is a method by which a suitable figure with two or more pairs of parallel sides is modified so that the design on one side is translated onto the parallel side. The result is a shape that will fit together into a complete design that also shows translational symmetry. Tiling by half-turn is a method that can be applied to a triangle. One half of one side of the triangle is modified, then the design is copied onto the remaining half of the same side by rotating the design one half turn about the midpoint of the side. In their tessellation designs, I did require that students modify each side of whichever initial shape they decided to try, but the level of the modification was left up to them. Some simply added a curved design or a simple tab to each side, while others really used their imagination to design pieces that resulted in Escher-like tessellations, as evident in the following student works.

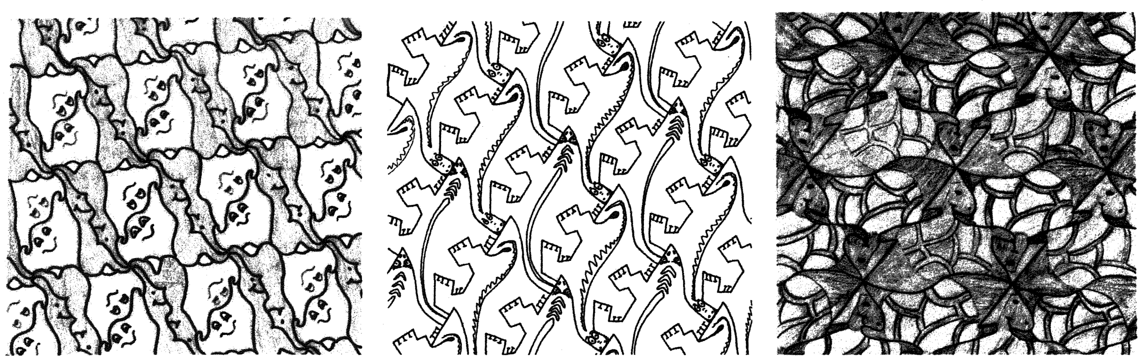


Figure 7. Student Tessellations

Perspective. The last large project unit that we covered dealt with the idea of perspective. The first activity in this unit allowed students to construct for themselves a two-dimensional image of a three-dimensional view. After discussing what "perspective" meant, I took the students to the atrium in our Science building, an area in the center of the building with five large windows on either side. Out of each set of windows, one wing of the building was in view. Students were divided into groups and stationed at one of the windows with a roll of masking tape. One student from each group was chosen to be the viewer; others were tapers. The directions were for the viewer to choose a place to stand from which he or she could see the wing of the building, then instruct the tapers to place tape on the window in an effort to construct a two-dimensional image of what the viewer could see.

This was an excellent exercise in teamwork as well as oral communication. The taping appeared to mimic the view only from the standpoint of the viewer. To the tapers, it appeared to be nothing more than a lot of tape on a window. Meanwhile, the viewer had to rely on oral communication to tell the others where to place the tape, since if the viewer moved out of position, the image would be lost.

The result of this activity with this class illustrated the importance of perspective in artwork. Each of the five groups stationed on one side of the atrium was taping the same wing of the building, yet the two-dimensional interpretations differed greatly based on where the groups' viewers were standing.

Figure 8. Students of one group taping the image seen by their designated viewer.

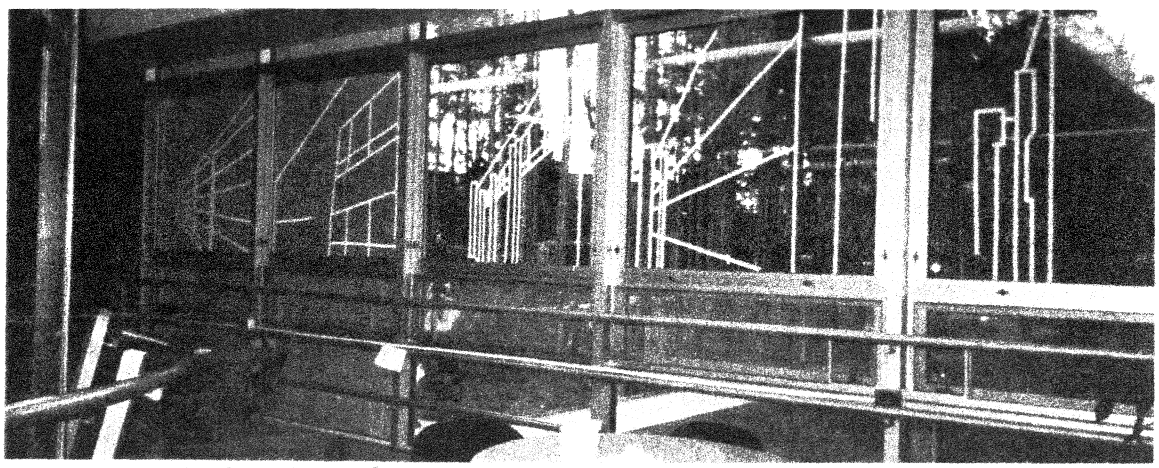


Figure 9. Five windows illustrating the different perspectives seen by the students.

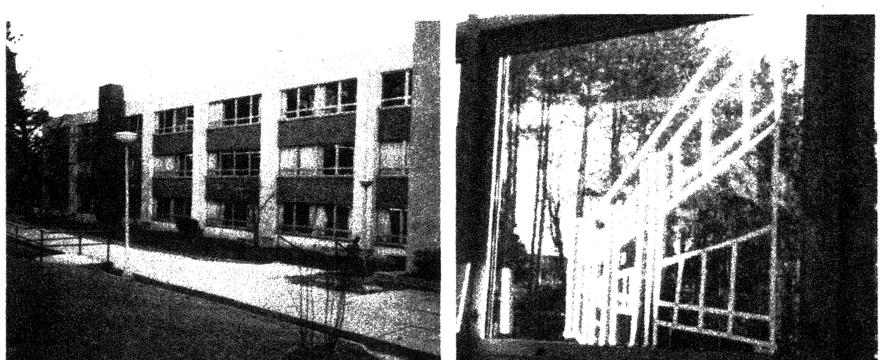
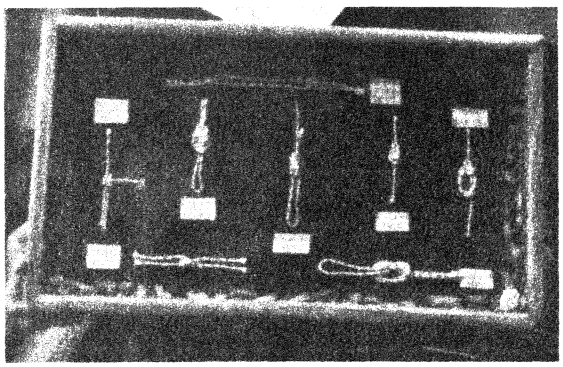
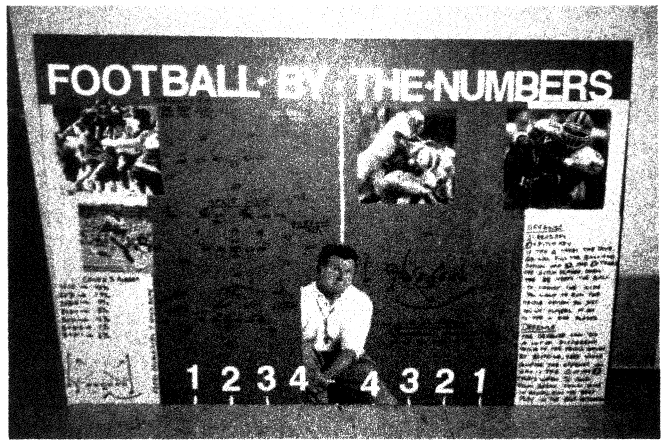


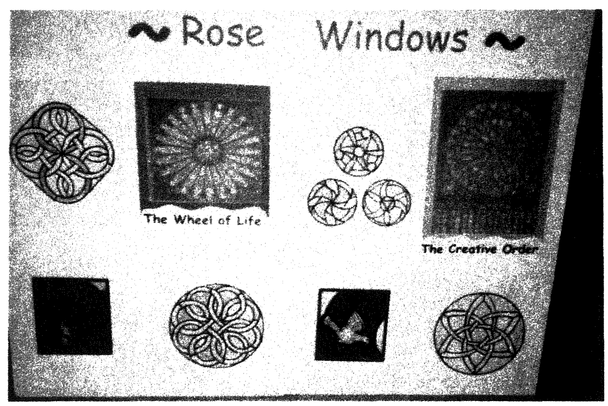
Figure 10. One group's perspective view of Peirce Science Center, left.

As a final exercise in the taping activity, students were required to attempt to locate the exact viewing position of the original viewer. Students learned that though they do not appear to be parallel when projected onto a two-dimensional plane, all lines that are parallel to the direction of view in the three-dimensional world meet at a point called the vanishing point. If a line perpendicular to the two-

Final projects. The semester culminated with the final exam. Students were allowed to choose any topic they wished; the requirement was simply to satisfactorily show the mathematical significance of the topic. Students had the option of producing a poster or creating a project. Exams were graded on originality, appearance, effort, and demonstration of mathematical significance. The variety and quality of the work were terrific. Sports, cooking, origami, the Hebrew alphabet and numerology, Poinsot stars, mobiles, knotboards, set design for a theatre production, kaleidoscopes, quilts, and tangrams are just a few of the outstanding posters or projects that were presented.







Concluding Remarks

What seemed like a semester of fun and games to my students seemed very different to me. They saw drawing and designing and taping, with little emphasis on mathematics. I saw them redrawing, redesigning, and retaping as they used the mathematics that they had learned without realizing what they were doing. On the first day of class, I gave a questionnaire asking the students what they expected to learn in this class. Most told me that they expected another traditional mathematics class, with formulas and equations and variables and word problems. But in the exit interview questionnaire this semester, 97% of the students completing the course said that this was one of the best mathematics courses they'd ever taken – that it had opened their eyes to mathematics in the world around them, and that they now could see that mathematics is a necessary and very much overlooked part of their everyday lives.

Student comments from the exit interview:

> "The fact that everything has some sort of math in it is quite eye-opening." – Senior, Political Science Major
>
> "This course has changed my outlook because it showed me that mathematics is everywhere you look!" – Sophomore, Special Education Major
>
> "Yes! I finally don't hate math!" – Senior, English Major
>
> "My outlook of math is very different now. The question always arises – what am I going to do with mathematical equations? Well, after taking this course I can create many things and explain many things through mathematics." – Freshman, Music Major
>
> "Well, I still despise mathematics – but I see it as more of a liberal lifelike subject rather than the cold, oppressed machine as I previously perceived it to be." – Freshman, Secondary Education/Social Studies Major
>
> "It really hasn't changed the way I feel about math – I still don't like it. This course just made it fun and helped me better understand things. This was the best math class I've ever taken!" – Senior, Communications Major
>
> "I am not math-oriented and in your course I actually learned (and dare I say had some fun!)" – Senior, Communications Major

I tell my students as they leave my class for the last time that if they can wake up every day for the rest of their lives and look at an object and see just one mathematical property in it, then my job is done. They don't have to tell me they like mathematics when they leave my class, but I want them to appreciate its existence. If I can do that, then I've met the challenge.

References

[1] COMAP. *For All Practical Purposes*, 4th Edition. New York : W. H. Freeman and Company. 1997. pp. 820, 825.