BRIDGES Mathematical Connections in Art, Music, and Science

## Weaving beyond Fibonacci

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#### Abstract

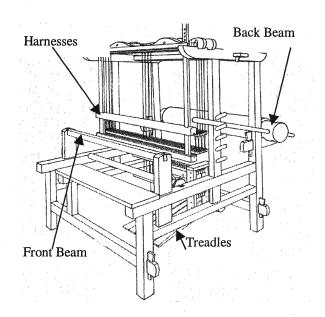
Weavers often use the Fibonacci sequence for number of threads in strips. Is there a reason for using this mathematical object? What other familiar sequences do weavers use? Is Fibonacci the only "beautiful" sequence?

### Conception

Weaving was one of the first practical arts. And so was mathematics. The chance to combine both came when a friend showed me her daughter's prayer shawl and stated that she had never had one. Since Hebrews are forbidden graven images, a rainbow is a perfect motif. Because colors in a rainbow blend, mathematical sequences could be used for blended effect. But, which mathematical sequence would be pleasing to the eye?

#### Weaving terminology

Weaving is done on a loom with thread or yarn. A loom is a structure to hold the yarn at the proper tension for making cloth. Thread on the loom is called warp, while the thread it is woven with is called weft. A single warp thread is tied to the apron fixed to the warp beam, passes over the front beam, through a reed in the beater, through a heddle on a harness (or shaft) and tied onto an apron fixed on the back beam (figure 1). When there are many threads on a loom, I can weave by pressing down on a treadle with a foot to raise all of the harness tied to the treadle creating a shed with some warp threads up and some at rest. A shuttle, containing a bobbin of weft thread, is thrown under the threads up and over the threads at rest, thereby weaving fabric.



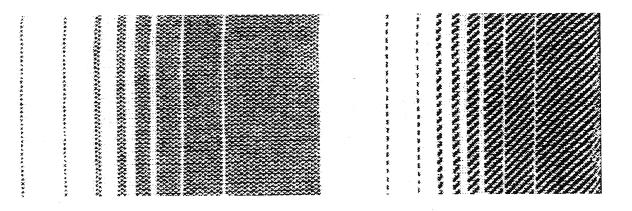
Pattern can be developed in four ways. Different weights of thread in the warp, with one weight of yarn in the weft, give a variation in the fabric woven. Yarn can be threaded on the harnesses in a repeated pattern and when woven, results in a texture that is not plain weave. Plain weave is threaded 1-2-3-4, the numbers being the numbers of the harnesses on which the yarn is threaded. This is called straight draft. Plain weave is treadled 1-3 together and then 2-4 together. Another way to vary the texture is to vary the treadling. If a loom is treadled 1-2, 2-3, 3-4, 4-1 with a weft shot for each set of treadles, it is called twill weave. The last way to develop a pattern is with color.

### Weaving plans

A good weaver weaves a sample before weaving fabric. This is to check sett or how many threads per inch and to check if the treadling pattern works with the colors. I planned on cotton yarn, mostly because the friends live in Arizona. Next is a check of the literature to see what sequences are used. None were mentioned but Fibonacci and random. Use of a sequence for 2-4-6-8, there would be two threads of blue, four threads of green, six threads of blue and eight threads of purple. While the fabric was to be woven in color, the sample was woven in black and white for reproduction purposes and woven both in plain weave and twill for comparison. Finally, I wove the sequences in the weft, with the warp being white as rewrapping the loom every six inches was counter productive.

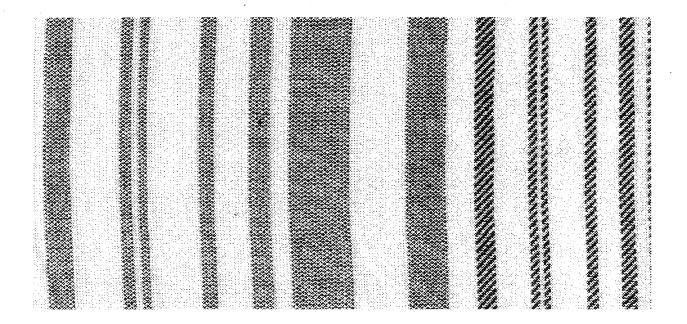
#### **Mathematical Sequences**

Fibonacci is a well-known sequence.



**Fibonacci Numbers:** Let  $F_1 = 1$ ,  $F_2 = 1$ ,  $F_{n+1} = F_n + F_{n-1} \{1, 1, 2, 3, 5, 8, 13, ...\}$ 

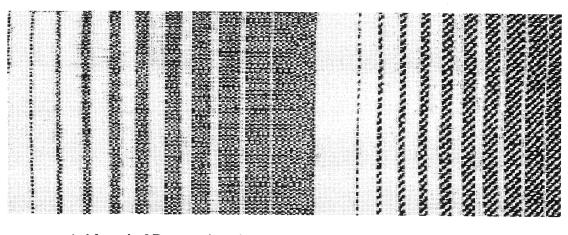
A random sequence for the weaving was determined by one of Plato's solids, an icosahedron, better known as a twenty-sided die.



#### **Random Numbers:** (1D20) {4, 9, 16, 4, 2, 3, 17, 5, 11, 7, 6, 19, 18, 4}

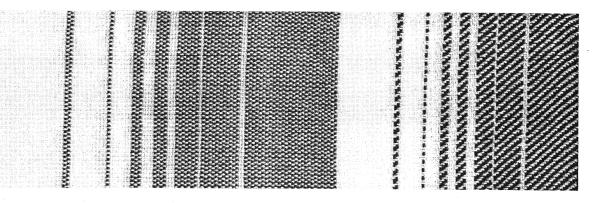
Since no other sequences were mentioned, I needed more possibilities. The sequences had to have no zeros, be low in overall numbers and be easily remembered by non-math oriented people.

The set of first ten integers is a simple sequence.



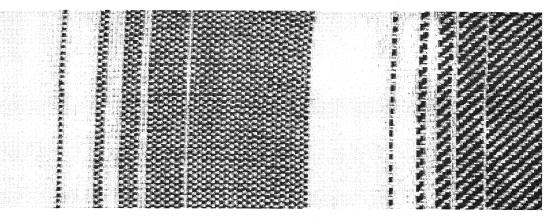
Arithmetical Progression:  $\{A_1 = 1, A_2 = 2, ..., A_n = n, ...\} \{1, 2, 3, ...\}$ 

Lucas numbers are similar to Fibonacci numbers, except the initial term is two not one.



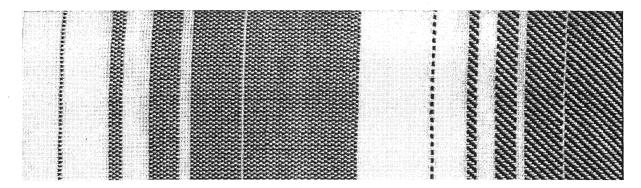
**Lucas:** Let  $L_1 = 2$ ,  $L_2 = 1$ ,  $L_{n-1} = L_n + L_{n+1}$ , {2, 1, 3, 4, 7, 11, 18, ...}

Powers of two is a sequence that was known to the Egyptians.



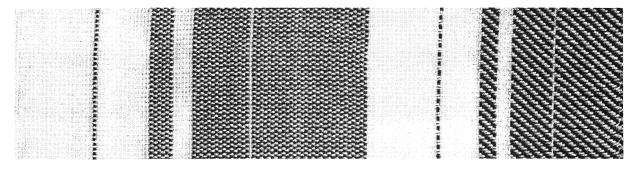
**Power of 2:**  $2^0 = 1, 2^1 = 2, 2^2 = 4, \{2^n \text{ where } n \in N\} \{1, 2, 4, ...\}$ 

Squares are a sequence that we all learn with our multiplication tables.



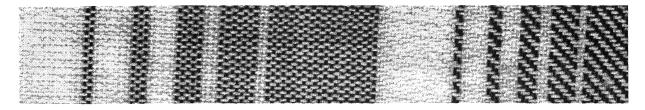
**Square Numbers:**  $S_1 = 1^2$ ,  $S_2 = 2^2$ ,  $S_3 = 3^2$ , ...,  $S_n = n^2$ , ...  $\{1, 4, 9, ...\}$ 

For Pascal's Triangle, I picked the sixth row.



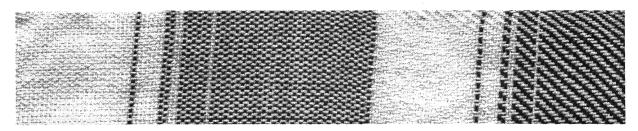
**Pascal's Triangle 6<sup>th</sup> Row:** {1, 6, 15, 20, ...}

Prime numbers are not a proper sequence but are well-known integers.



**Prime Numbers:** {2, 3, 5, 7, 11, ... }

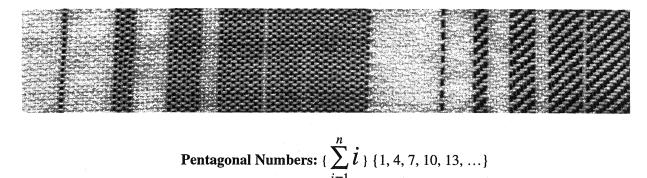
A lesser- known sequence is that of factorials.



**Factorials:**  $1! = 1, 2! = 2, 3! = 6, 4! = 24, ..., n!, ... \{1, 2, 6, 24, ...\}$ 

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And the final sequence was that of pentagonal numbers.



# Conclusion

Just as we want our music to have pattern, we want our fabric used for clothing to have pattern. Fabric used for art may have fewer patterns rather like surrealist art has harder to find patterns. So any of the above sequences should be used for fabric.

#### Works in Progress

As of this writing, I have a scarf in three shades of blue using squares sequence in plain weave on my loom. I am chaining the warp for the prayer shawls with the rainbows in Fibonacci sequence to be woven in mixed plain weave and twill. I am planning a green, blue and purple progression for vest fabric in Lucas numbers for a third project. As a matter of complication, the vest will be woven in undulating twill.

#### References

Sharon Alderman, *The Fibonacci Series*, Handwoven, Volume IV. Pp 45, 1983 Joan Michaels-Paque, *Syllabus for Success*, Shuttle, Spindle&Dyepot, Volume XV, pp 11-13, 1984 Jennifer Moore, *The Golden Proportion*, Handwoven, Volume XXI, pp. 64-67, 2000 Lana Schneider, *Algebraic Expressions: Designs for Weaving*, Handwoven, Volume XIX, pp. 48-51, 1998

N. J. A. Slone, Handbook of Integer Sequences

Edward F. Worst, Weaving with Foot Power Looms, Dover, 1974