BRIDGES
Mathematical Connections
in Art, Music, and Science

Mathematics and Art: Bill and Escher

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Introduction

Mathematicians pay considerable attention to the aesthetic qualities of their discipline; this explains why many of them think that mathematical and artistic activities are to some extent very similar, or comparable. Such an attitude also explains in part why mathematicians have periodically not only attempted to explain the fascinating beauty of their discipline, but also to interpret and analyze art through mathematics. On the other hand, there are many cases of artists, from different periods, who have been fascinated by mathematics and have tried to use its ideas and techniques in their work. And some artists have indeed been forced to become mathematicians, as happened during the Renaissance.

Renaissance painters turned to mathematics not only because they had the problem of depicting the natural world realistically on canvas, of producing scenes in three dimensions with depth, but also, as Morris Kline has pointed out in his important book on mathematics in western culture [1], they were profoundly influenced by the rediscovery of Greek philosophy. They were wholly convinced that mathematics was the true essence of the physical world and that the universe was ordered and explainable in geometric terms. This great interest forced Renaissance painters to become – as Kline defined them – the best applied mathematicians of the period. Since the professional mathematicians of that time did not have the geometric instruments which the artists needed, they themselves also had to become learned and active theoretical mathematicians.

"Even though there still exist some pseudo-humanists who are proud of their non-comprehension of mathematics, the growing number of laymen who regret not being able to take part fully in this banquet of the Gods is quite reassuring." This is the opening of the preface to the book *Les Grands Courants de la Pensée Mathématique*, the preface of which was written by the mathematician François Le Lionnais [2]. The aim of the book was to attempt to show "not the immobile panorama of the sectors pertaining to mathematics, but first and foremost the direction towards which the diverse mathematical disciplines are moving." In the chapter on *Arts et Esthétique: Les Mathématiques et la Beauté*, Le Lionnais replies to those who would reduce the relationship between mathematics and art to a question of proportions and numbers: "In mathematics there exists a beauty which must not be confused with the possible influence of mathematics on the beauty of the works of art. The aesthetics of mathematics must be clearly distinguished from the applications of mathematics to aesthetics."

The words used by Le Lionnais to describe the beauty of mathematics are no different from those used by many other mathematicians: "And so, beauty shows itself in mathematics just as it shows itself in the other sciences, in the arts, in life and in nature. While at times comparable to pure music, to great painting or poetry, the emotions that mathematics arouse are of a different nature that cannot be understood unless one has had a sort of illumination within oneself. The beauty of mathematics is certainly no guarantee of truth or usefulness. But it enables some people to enjoy a unique experience, while for others it provides the certainty that mathematics will continue to be practiced for the benefit of everyone and for the glory of the human adventure."

I would like to mention two artists who have attracted my interest in the study of a possible relationship between art and mathematics: Max Bill and M. C. Escher. I have made documentary films about both artists' works. I worked directly with Bill, while in Escher's case, I used his works without ever having met him. I will try to point out several features that these two artists have in common [3], [4].

Max Bill's mathematical approach to art

There is no doubt that the clearest approach to the possibility of a mathematical approach to the arts has been formulated by the famous Swiss artist Max Bill. In 1949 he wrote: [5] "By a mathematical approach to art, it is hardly necessary to say I do not mean any fanciful ideas for turning out art by some ingenious system of ready-reckoning with the aid of mathematical formulas. So far as composition is concerned, every former school of art can be said to have had a more or less mathematical basis. Even in modern art, artists have used methods based on calculation, inasmuch as these elements, alongside those of a more personal and emotional nature, give balance and harmony to any work of art."

These methods had become more and more superficial, for the artist's repertory of methods had remained unchanged, except for the theory of perspective, since the days of ancient Egypt. The innovation occurred at the beginning of the twentieth century: "It was probably Kandinsky who gave the immediate impulse towards an entirely fresh conception of art. As early as 1912... Kandinsky in his book *Ueber das Geistige in der Kunst* [6] indicated the possibility of a new direction which would lead to the substitution of a mathematical approach for improvisations of the artist's imagination... It is objected that art has nothing to do with mathematics; that mathematics, beside being by its very nature as dry as dust and as unemotional, is a branch of speculative thought and as such in direct antithesis to those emotive values inherent in aesthetics... yet art plainly calls for both feeling and reasoning."

We must not forget that Max Bill was first and foremost a sculptor who believed that geometry, which expresses the relations between positions in the plane and in the space, is the primary method of cognition, and can therefore enable us to apprehend our physical surroundings, so, too, some of its basic elements will furnish us with laws to appraise the interactions of separate objects, or group of objects, one to another. And again, since it is mathematics that lends significance to these relationships, it is only a natural step from having perceived them to desiring to portray them. Visualized presentations of that kind have been known since antiquity, and they undoubtedly provoke an aesthetic reaction in the beholder. In the search for new formal idioms expressive of the technical sensibilities of our age, these borderline exemplars had much the same order of importance as the "discovery" of native West African sculpture by the Cubists.[7]

And here is the definition of what must be a mathematical approach to the arts: "It must not be supposed that an art based on the principles of mathematics, such as I have just adumbrated, is in any sense the same thing as a plastic or pictorial interpretation of the latter. Indeed, it employs virtually none of the resources implicit in the term "pure mathematics". The art in question can, perhaps, best be defined as the building up of significant patterns from the ever-changing relations, rhythms and proportions of abstract forms, each one of which, having its own causality, is tantamount to a law unto itself. As such, it presents some analogy to mathematics itself where every fresh advance had its "immaculate conception" in the brain of one or other of the great pioneers.

Thus Euclidean geometry no longer possesses more than a limited validity in modern science, and it has an equally restricted utility in art. To convince his readers, after having clarified his thoughts, Bill needed to provide some examples which pertained to his point of view as an artist – examples of what he called "the mystery enveloping all mathematical problems", "the inexplicability of space – space that can stagger us by beginning on one side and ending in a completely changed aspect on the other, which somehow manages to remain that self-made side; the remoteness or nearness of infinity – infinity which may be found doubling back from the far horizon to present itself to us as immediately at hand; limitations without boundaries; disjunctive and disparate multiplicities constituting coherent and unified entities; identical shapes rendered wholly diverse by the merest inflection; fields of attraction that fluctuate in strength; or, again, the space in all its robust solidity; parallels that intersect; straight lines untroubled by relativity, and ellipses which form straight lines at every point of their curves.

For though these evocations might seem only the phantasmagorical figments of the artist's inward vision, they are the projections of latent forces... Hence all such visionary elements help to furnish art with fresh content. Far from creating a new formalism, what these can yield is something far transcending surface values since they not only embody form as beauty, but also form in which intuitions or ideas or conjectures have taken visible substance.

It may be contended that the result of this would be to reduce art to a branch of metaphysical philosophy. ... I assume that art could be made a unique vehicle for the direct transmission of ideas because, if these were expressed by pictures or plastically, there would be no danger of their original meaning being perverted by whatever fallacious interpretations. Mental concepts are not as yet directly communicable to our apprehension without the medium of language; though they might ultimately become so by the medium of art.

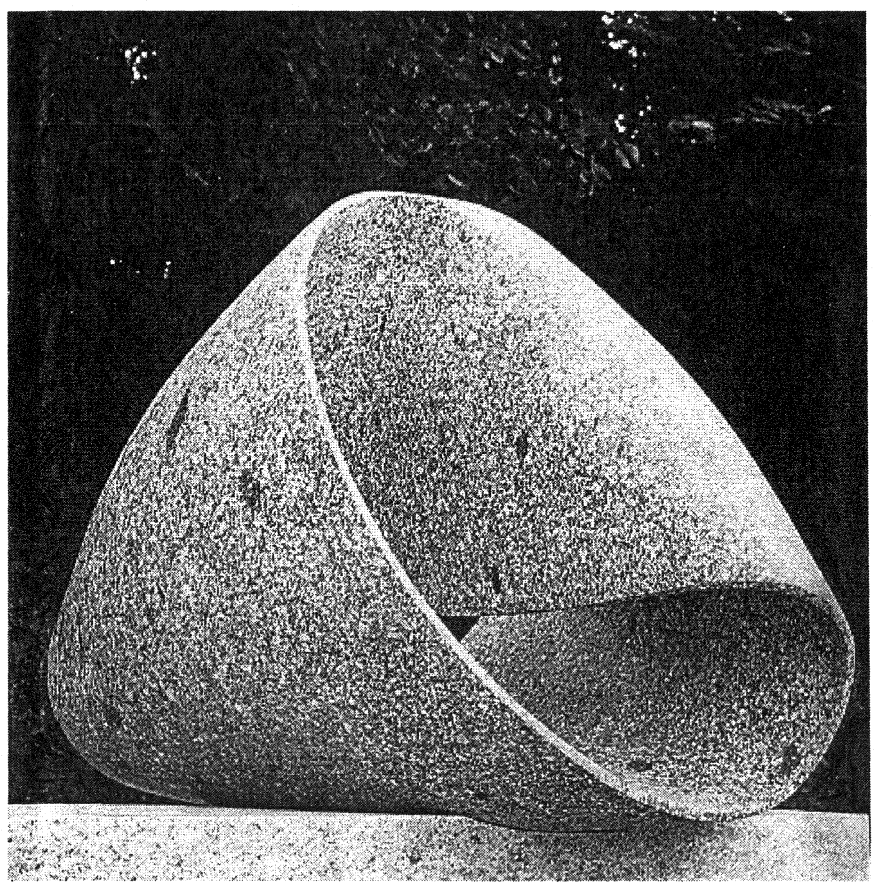


Figure 1: *Max Bill, Endless ribbon, granite, 1935-1953.*

Thus the more succinctly a train of thought was expounded and the more comprehensive the unity of its basic idea, the closer it would approximate to the prerequisites of the mathematical way of thinking. The orbit of human vision has widened and art has annexed fresh territories that were formerly denied to it. In one of these recently conquered domains, the artist is now free to exploit the untapped resources of that vast new field of inspiration. And despite the fact that the basis of this mathematical way of thinking in art is in reason, its dynamic content is able to launch us on astral flights which soar into unknown and still uncharted regions of the imagination."

This is very clear praise by a great artist for the artistic quality of certain aspects of mathematics. When mathematicians think about the beauty of mathematics, they generally have in mind examples of a type that are not comprehensible to non-specialists. However, the artist states that not only does mathematics provide aesthetically relevant models, but also that it has deep cultural influence. And this is why a mathematical route to art is possible.

One of the visual ideas that Max Bill used without realizing it was the Moebius ribbon. Max Bill called some of his sculptures *Endless Ribbons* and in fact they are shaped like Moebius ribbons. In his article *How I began to make single-faced surfaces*, Bill recounts how he discovered the Moebius surfaces [8]: "Marcel Breuer, my old friend from the Bauhaus, is the real originator of my single-faced sculptures. This is how it happened: in 1935 in Zurich, together with Emil and Alfred Roth, I was building the Doldertal houses which in those days were much talked about. One day, Marcel told me he had been commissioned to design a house for an exhibition in London. It was to be the model of a house in which everything, even the fireplace, would be electric. It was obvious to all of us that an electric fireplace, which glowed without flames, would not be a particularly attractive object. Marcel asked me if I would like to make a piece of sculpture to be hung above it. I began looking for ideas – a structure that could be hung over a fireplace and which might even turn in the upward flow of hot air. With its shape and movement, it would in a sense act as a substitute for the flames. Art instead of fire! After many experiments, I came up with a solution that seemed reasonable."

The interesting thing to note is that Bill thought he had invented a completely new shape. Even more curious was that he discovered it by twisting a strip of paper, just as Moebius had done many years previously.

"Not long afterwards, people began to congratulate me on my fresh and original interpretation of the Egyptian symbol for infinity and the Moebius ribbon. I had never heard of either of them. My mathematical knowledge had never gone beyond routine architectural calculations, and I had no great interest in mathematics."

Max Bill's *Endless Ribbon* was put on display for the first time at the Milan Triennale exhibition in 1936. "Since the 1940s – wrote Bill – I had been thinking about problems of topology. From them I developed a sort of logic of shape. There were two reasons why I kept on being attracted by this particular theme: 1) the idea of an infinite surface – which was nevertheless finite – the idea of a finite infinity; 2) the possibility of developing surfaces that – as a consequence of the intrinsic laws implied – would almost inevitably lead to shapes that would prove the existence of the aesthetic reality. But both 1) and 2) also indicated another direction. If non-oriented topologic structures could exist only by virtue of their aesthetic reality, then, in spite of their exactness, I could not have been satisfied by them. I'm convinced that the basis of their efficacy lies in part in their symbolic value. They are models for contemplation and reflection."

Bill had the idea of setting up a room for his topological sculptures in the topological section of the permanent mathematics exhibition in London's Science Museum. Unfortunately, the project was never carried out.

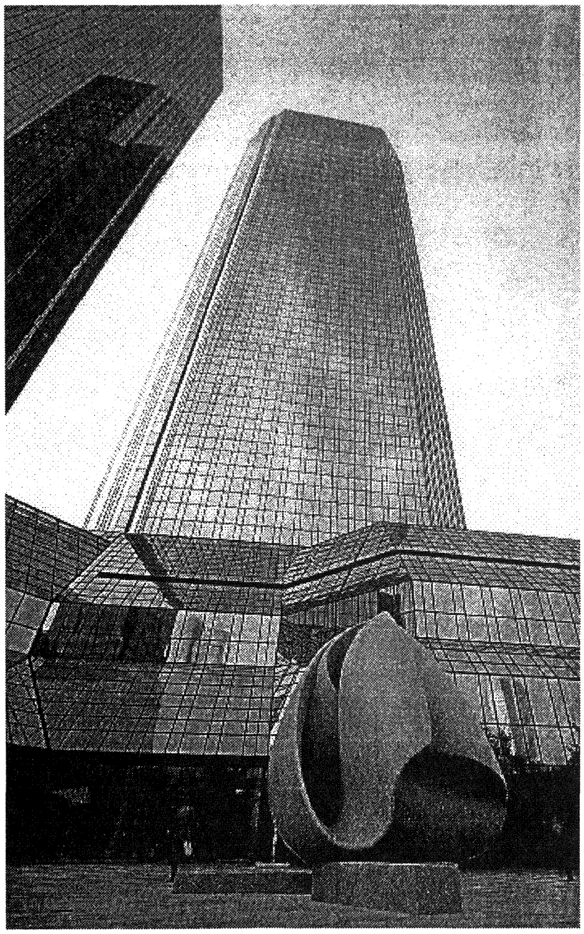


Figure 2: *Max Bill, Kontinuität, granite, 1986; from the book W. Spies, Kontinuität. Granit-Monolith von max Bill, Deutsche bank, 1986.*

The graphic artist M.C. Escher (1898-1972)

The Dutch artist Maurits Cornelis Escher was born in 1898. Escher's tale is quite unusual: for a long time, his work was almost completely unknown and unappreciated. After almost 20 years spent in Italy, he had shown only a handful of times. At a certain point in the Sixties, however, his fame began to grow, among scientists, mathematicians, physicists, and crystallographers. The history of the relationship of Escher with scientists, mathematicians especially, is quite interesting in terms of understanding how the Dutch artist thought of his work. Escher defined himself, and not unreasonably, a graphic designer. The fundamental event in his life, he said, was in 1938, when the Escher family had already left Italy after a long stay. "In Switzerland, Belgium and Holland... I found the landscape and architecture much less interesting than in Southern Italy. I felt driven to distance myself ever more from more or less direct and realistic illustration of the surrounding reality. There is no doubt that these unusual circumstances were responsible for my having brought to light these 'interior visions.'"

All the illustrations in his first book, except for the first seven works, were done with the idea of communicating a detail of these interior visions. "The ideas that are basic to them often bear witness to my amazement and wonder at the laws of nature which operate in the world around us. He who wonders discovers that this is in itself a wonder. By keenly confronting the enigmas that surround us, and by considering and analyzing the observations that I had made, I ended up in the domain of mathematics. Although I am absolutely without training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists." [9]

These last two comments by Escher summarize perfectly the privileged relationship that the artist established with the scientific community. That Escher himself thought of scientists as his privileged audience is beyond doubt.

The title of the first monographs on Escher "The world of M.C. Escher" [10] is very appropriate. Escher created his own world of meticulously constructed images; a world at once fantastic, imaginative and magic but also realistic, coherent and detached; observed with an eye apparently lacking in emotion. Escher was an artist of minute details, tiny elements that create instability and are disturbing the apparent calm of the whole.

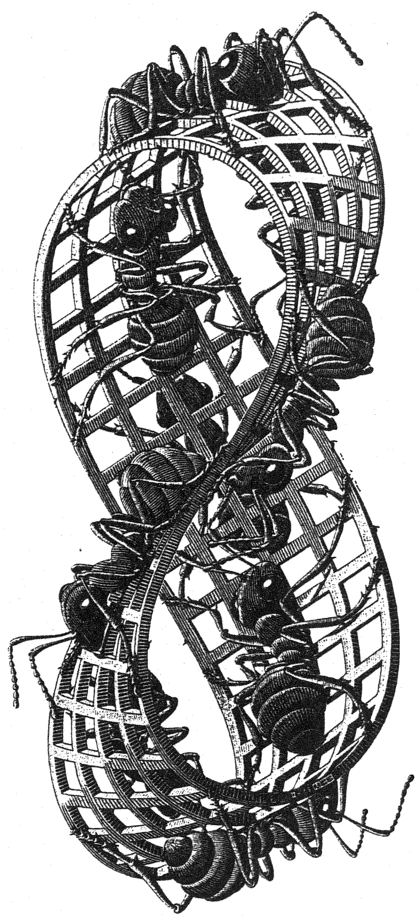


Figure 3: *M.C. Escher, Moebius Strip II, wood engraving, 1963;by the kind permission of the M.C. Escher Foundation. Baarn. Holland. All M.C. Escher works © Escher Heirs c/o Cordon Art, Baarn, Olanda. All right reserved*

The first reason for using a movie camera to film Escher's prints is that cinematic technique first of all lets us concentrate the attention on what the filmmaker wants to be looked at. The film forces us to look at Escher's world as if we were part of it. We are not distracted by anything. The second reason is that Escher's drawing technique is very precise and detailed. Many of these details are very minute despite being very precisely drawn. Cinema technique allows us both to isolate these details and also magnify them many times in order to appreciate from close up the precision of the artist's method. Many of Escher's works are like a story that develops. They must be observed in the sequence suggested by Escher himself. This is why the movie camera permits a very precise and accurate analysis of Escher's works.

A very significant example, which sums up the various aspects, is the "Metamorphoses"; in particular "Metamorphosis II" which immediately seemed to me a perfect cinematic sequence. I have used the animation of this Escher's work as the final sequence of the video.

Escher made the following comments to this work: [11]: “First the black insect silhouettes join; at the moment when they touch, their little white background has become the shape of a fish. Then figures and background change places and white fish can be seen swimming against a black background. A succession of figures with a number of metamorphoses acquires a dynamic character. Above I pointed out the difference between a series of cinematographic images projected on a screen and the series of figures in the regular division of plane. Although in the latter the figures are shown all at once, side by side, in both cases the time factor plays a role. ”

In 1964 Escher was visiting his son George Escher in Canada. He was invited by several organizations in the USA, including the MIT and the Bell Laboratories, to give presentations on his work. Shortly after arriving in Canada, Escher had to be admitted to Saint Michael's hospital in Toronto for an emergency operation. All appointments had to be canceled, and Escher would never again have a chance to give his carefully prepared lectures.

Escher had written the complete English text of his lectures and these texts have been preserved. In a book published in 1989 "Escher on Escher: exploring the infinite"[12] all these materials have been published. The chapter is entitled: "The lectures that were never given". The final part of his talk was dedicated to "Metamorphose II": “I propose to round off this talk by showing you a woodcut strip with a length of thirteen feet. It's much too long to display in one or even in two slides, so I had it photographed in six parts, which I can present in three successive pairs and which you are invited to look at as if it were one uninterrupted piece of paper. It is a picture story consisting of many successive stages of transformations. The word "Metamorphose" itself serves as a point of departure. Placed horizontally and vertically in the plane, with the letters O and M as points of intersection, the words are gradually transformed into a mosaic of black and white squares, which, in turn, develop into reptiles. If a comparison with music is allowed, one might say that, up to this point, the melody was written in two-quarter measure.

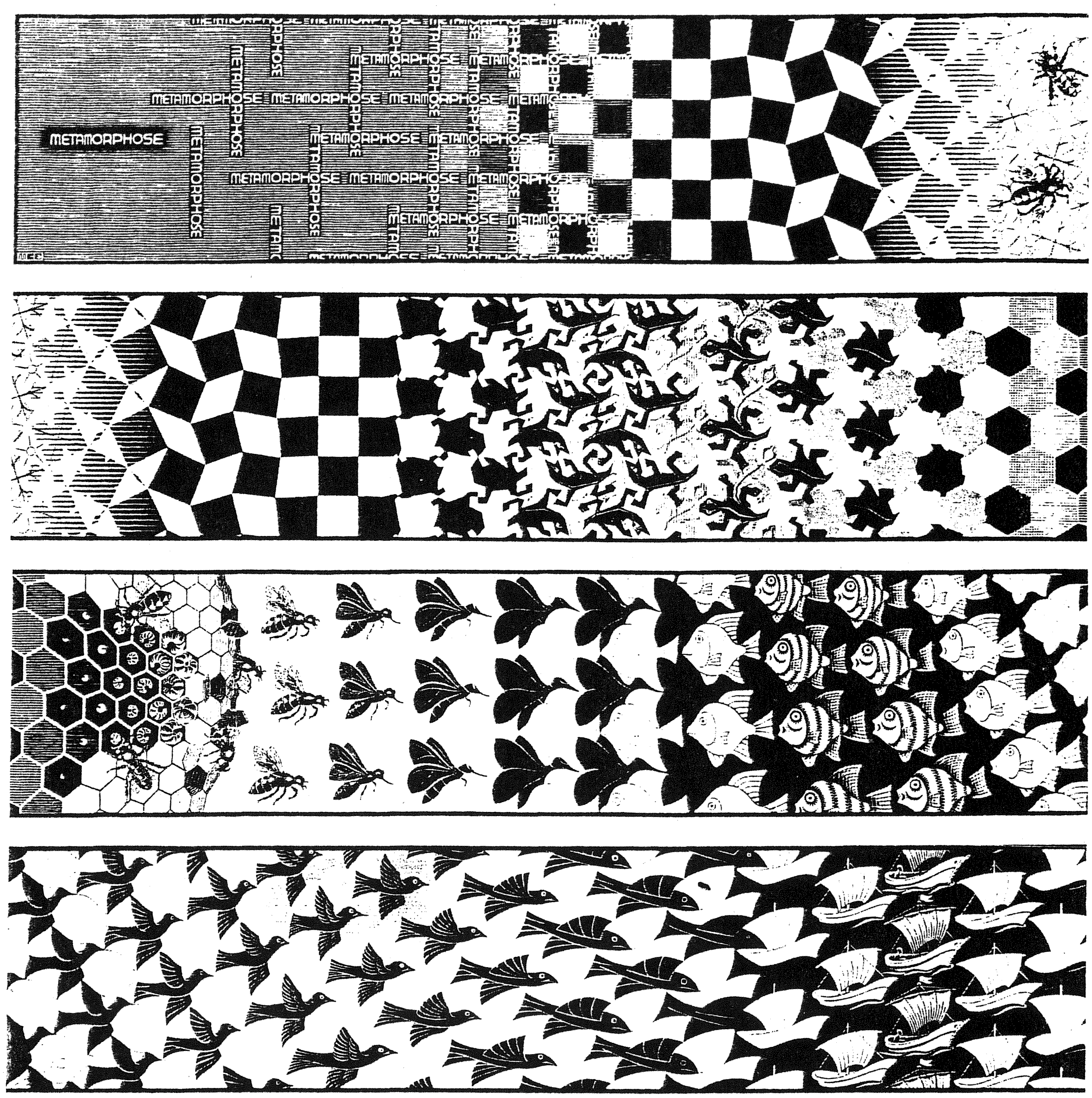
Now the rhythm changes: bluish elements are added to the white and black, and it turns into a three-quarter measure. By and by each figure simplifies into a regular hexagon. At this point an association of ideas occurs: hexagons are reminiscent of the cells of a honeycomb, and no sooner has this thought occurred than a bee larva begins to stir in every cell. In a flash every adult larva has developed into a mature bee, and soon these insects fly out into space.

The life span of my bees is short, for their black silhouettes soon merge to serve another function, namely, to provide a background for white fishes. These also, in turn, merge into each other, and the interspacing takes on the shape of black birds. Then, in the distance, against a white background, appear little red-bird silhouettes. Constantly gaining in size, their contours soon touch those of their black fellow birds. What then remains of the white also takes a bird shape, so that three bird motifs, each with its own specific form and color, now entirely fill the surface in a rhythmic pattern.

Again simplification follows: each bird is transformed into a rhomb, and this gives rise to a second association of ideas: a hexagon made up of three rhombs gives a plastic effect, appearing perceptively as a cube. From cube to house is but one step, and from the house a town is built up. It's a typical little town of southern Italy on the Mediterranean, with, as commonly seen on the Amalfi coast, a Saracen tower standing in the water and linked to shore by a bridge. (It is the town of Atrani)

Now emerges the third association of ideas: town and sea are left behind, and interest is now centered on the tower: the rock and the other pieces on a chessboard. Meanwhile, the strip of paper on which "Metamorphose" is portrayed has grown to some twelve feet in length. It's time to finish the story, and this opportunity is offered by the chessboard, by the white and black squares, which at the start emerged from the letters and which now return to that same word "Metamorphose".

This was the end of the lecture of Escher. It is a story he was telling based on his woodcut "Metamorphose". When I was making my video I was not aware of the text of this lecture of Escher. But my idea was exactly the same described by Escher, perhaps more: not only a story but what in cinema is a story-board, a precise description of a sequence to be filmed, made usually by drawings and words. So I have used Escher's original engraving as a storyboard for making the animation of "Metamorphose". What Escher has to described by words, because it was impossible to show the complete work to the audience, it is described in the movie without any words: just the animation, following the time of changing of the various forms in the woodcut, and music, another suggestion of Escher himself! To paraphrase Escher we can say: In this film they are the images and not the words that come first. [13]



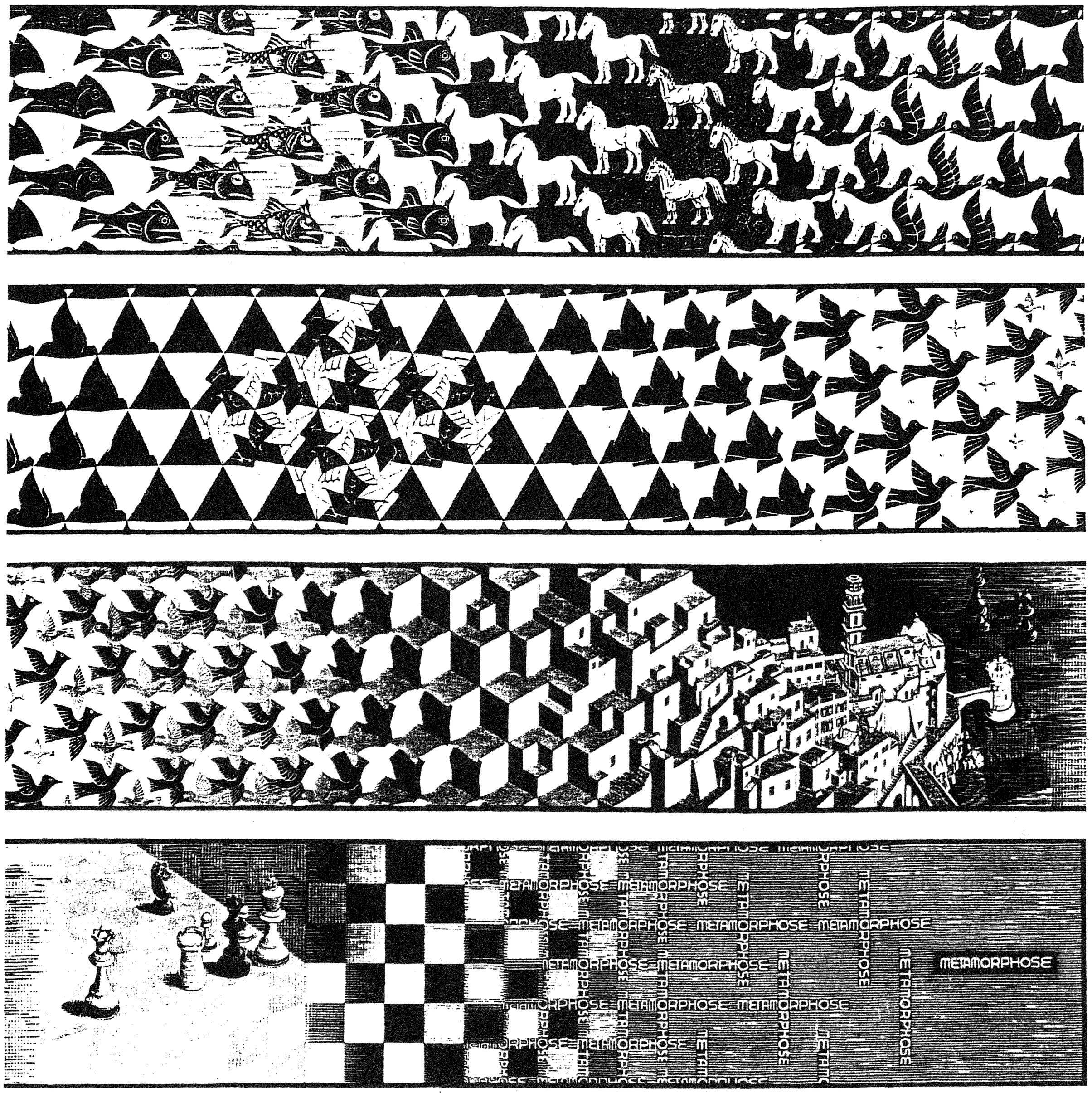


Figure 4: *M. C. Escher, Metamorphose, xylogravure, 1967/68. by the kind permission of the M.C. Escher Foundation. Baarn. Holland. All M.C. Escher works © Escher Heirs c/o Cordon Art, Baarn, Olanda. All right reserved*

Concluding remarks

A new phenomenon that appeared only a few years ago is modifying the quality of the relationship between mathematics and art. Even granting that mathematics is an art, and that it is possible to apply mathematical criteria in art, and thus to build a mathematical approach to art, art nonetheless remains something different from mathematics, and no mathematician expects to be remembered as a great artist. Thanks to the explosion of technology and the increasingly sophisticated techniques offered by computer graphics, over the past few years it has become possible to visualize mathematical phenomena whose existence could hardly have been otherwise supposed. "And since this science possesses these fundamental elements and puts them into meaningful relationships, it follows that such facts can be represented, or transformed into images...which have an unquestionably aesthetic effect." So wrote Max Bill in 1949. Do his words apply to the images created by mathematicians in recent years?

The new fact is that some mathematicians not only continue to claim that their discipline is an art, but also now wish to be thought of as artists in their own right. Over the past several years, there has been an enormous output of mathematical images, of objects resulting from extremely disparate mathematical studies. Some examples are highly interesting from both the strictly mathematical and aesthetic standpoints, at least according to the mathematicians who developed them. I do not mean at all to say that if it is possible to speak of progress in mathematical knowledge, then this must imply an idea of improvement in the domain of art, where an argument of this kind makes no sense. At any rate, we can expect further changes in the relationships between mathematics and art, and I may say so, most of the credit must go to the mathematicians. [14]

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