BRIDGES
Mathematical Connections
in Art, Music, and Science

The Poetics of Mathematics in Music

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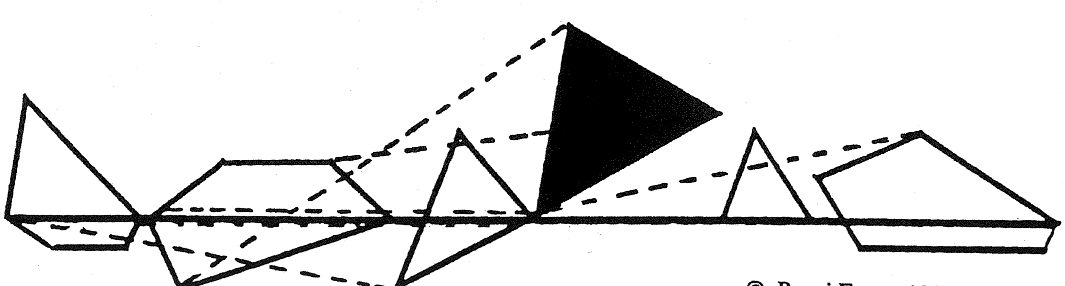
"In the same way that Plato conceived the Great Ordering One... As arranging the Cosmos Harmoniously according to the pre-existing..., so the..., neo-Platonic... view of Art conceived the Artist as planning (the) work of Art according to a pre-existing system of proportions (mathematics), as a symphonic composition, ruled by a dynamic symmetry corresponding in space to musical eurhythm time. This technique of correlated proportions was in fact transposed from the Pythagorean (-Theano) conception of musical harmony." So does the Introduction of Matila Ghyka's book, *The Geometry of Art and Life,* state on the very first page. Jacob Bronowski writes that, "discoveries of science and the works of art are both explorations and explosions of a hidden likeness". Hermann Weyl refers to "the loveliness hidden under the surface beauty of nature, that the mathematics is not to be revealed in its skin". The biologist Gregory Bateson mentions in his book, *Mind and Nature* that, "being responsive to the pattern which connects means developing the critical aspect and the esthetic experience."

The musics to be briefly discussed and for which mathematical models were developed, are strata of connective proportional patterns not just describing a nicety, yet involving a remarkable invention and recalling the ability to fulfill it. Furthermore, these connective proportional patterns are often "hidden under the surface" -- joy and challenge of the listening or reading experience. All the musics, from the early medievalist Hildegard von Bingen to the Eve (Africa), Kwakuilt and Zuni (North American Indians), and Piro (South American Indian), beyond Europe, to Guillaume Machaut, Franz Schubert, Frederic Chopin, Anton Von Webern, Ruth Crawford, Luigi Dallapicola, Milton Babbitt, and Gyorgy Ligeti, are the logic-processed numbers of that hidden structure where space and time are traced through accurate measurements by the composer on a small or large canvas of moving sonic elements. A chain of interrelated symmetries (asymmetries) produces the explored geometry behind the musical notation; the audition might indeed fail to grasp it and so too the first understanding; but the overall pattern -- a finished puzzle of small patterns -- is the ultimate constellation of that imagined sonic geometry.

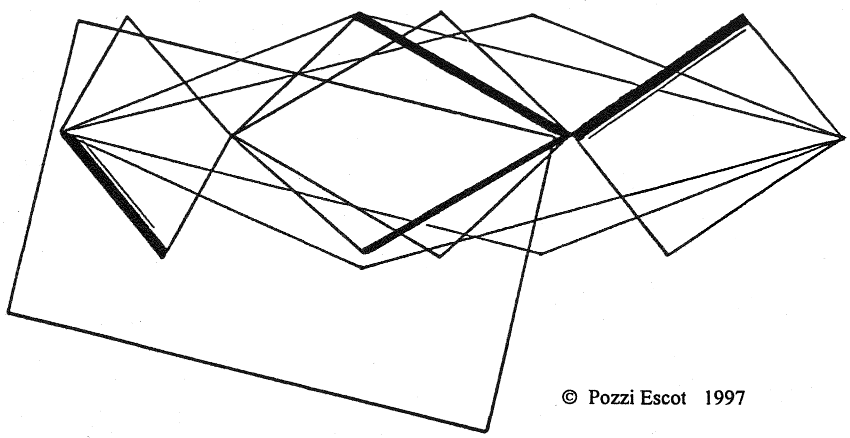
The French mathematician Jacques Hadamard writes in his book *The Psychology of Invention* of the creative process as one of preparation, incubation, illumination, and correction; and the English physicist-mathematician H. E. Huntley writes about what constitutes the essence of creativity: surprise (unexpected encounter), wonder (unexplored world), and curiosity (a craving to understand). Jung, in the development of his theory of the collective unconscious, writes of the secret of effective art, "who speaks with primordial images speaks with a thousand tongues". Is the creative action, with invention and results, guided principally by an energetic struggle towards a deep-seated order, consciously and openly dictated by a conglomerate of measurements? Or is it a fusion of both intuitive and mathematical spirit? Is it not correct to absorb the fact that structure is movable in reference to function?

If the answer is positive to both questions, as we try to confirm with these mathematical models (three examples follow), then cultures or historical periods might have similar parameters fostering the creative effort and process; yet these parameters may be expressed in entirely different designs without at all lessening the power of these parameters. It is known in physics that when the atoms are arranged in a three-dimensional molecule, their behavior can drastically be altered when they are removed from the determined shape with respect to each other. What logic then must govern the world of composition, anywhere, anytime? H. E. Huntley again writes that the likeness that we create is a sort of beauty and that this beauty must have a compendium of certain elements or parameters: alternation of tension and relief, realization of expectation, perception of unsuspected relationships, brevity, unity, joy, sense of wonder. Every bit of art and science has a respective reality -- a reality which obeys a training -- a training permeated with a timing and a spatial condition; and this is the reality which must be ultimately recovered and discerned. We must find the proportions and study the dimensions to discover the formalized, imagined, and often unique construction. We must map the relevant components in the composition of music, where a myriad of aspects is combined to conquer an architectural sonic design.

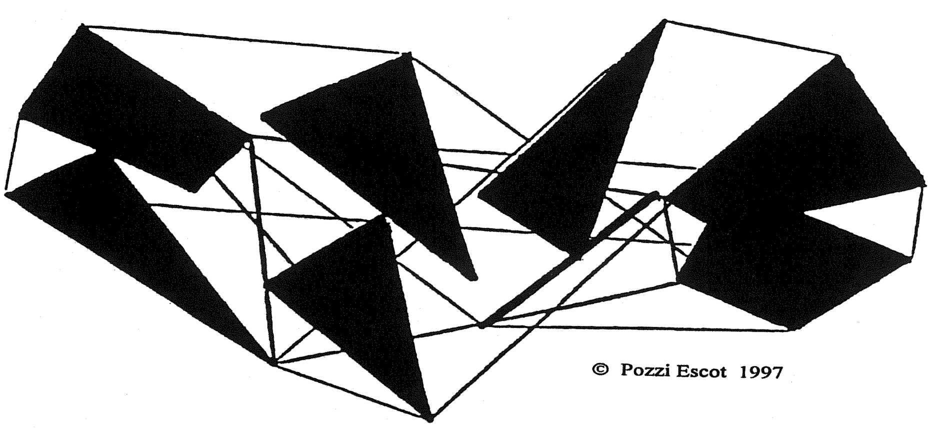
Example 1 shows a model based on the graphic notation of Hildegard von Bingen's antiphon "Sed diabolus" where the phrases' geometry is drawn taking four vertices (onset, apex, nadir, and decay pitches) as definition of its contour. The model clearly shows the striking removal of the climatic fourth phrase from the horizontal axis of the Final (the priority pitch defining the acoustical framework of the chant), and its peculiar surrounding of triangles. The composer has applied all three mathematical means of Boethius to the dimensions of the chant. The six phrases have 9, 16, 9, 12, 6, 16 equal attack points each obtaining: 6-9-12 arithmetical mean, 6-9-16 harmonic mean; 9-12-16 geometrical mean. The negative Golden Mean divides the antiphon exactly into two and four phrases (9+16; 9+12+6+16); the positive occurs in the extraordinary fourth phrase where the highest point of the antiphon is heard. The broken lines show the Golden Mean negative distances among precise vertices of the phrases' geometrical contours including the distance from the highest point to the first lowest point of the chant; the parallel (-dyad openings) of the first and fourth phrases dividing the antiphon into two exact halves of 34 points each (9+16+9; 12+6+16).



Example 2 takes all the Golden Mean parameters of the North American Kwakuitl "Raven Song" based on a complex plane translationof the music with the abscissa determining the temporal flow and the coordinate the frequency of the song (128 quarter value durations) times a total ambitus of 18 (each frequency division at a 100 cents.) The entire song presents a multiplicity of special geometrical mean divisions at salient events of the linear sound flow. This colossal plurality manifest a control of the song's balances not only in its minor-scale structure, but in its large-scale as well. The model presents vividly these proportions.



Example 3 shows a model of the composition "Duet" by the American composer Milton Babbitt. The model is based in a particular array of twelve-pitch pair-sets where: let A {a,b,c,d,e,f} + n be a first hexacord of different pitches of a Prime set and where n stands for the missing interval among the six pitches and where A+n obtain B the second hexacord of different pitches completing a chromatic collection with the first hexacord of a Prime set (P). The eight sets through their geometrical perimeters show a vivid symmetry both rotational and bilateral. All proportions are so accomplished as to provide a listening experience.



The mathematical models are the result of numerical information from analytical studies; information obtained from the parts' distinguishing qualities that make up the composition: ranges, intervals, pitches, attack points, durations, dynamics, values, vertices of onset/apex/nadir/decay in phrasial divisions, large-scale climax, motion. Often composers have deliberately applied mathematics, formulas like the mathematical means including the special Golden Mean, to these components of the compositions of music. Some of the models are simply graphic representations using a complex plane relating the coordinate as frequency and abscissa as duration; thus, another way of notating the music which vividly conveys a different perspective. Some of the models are geometrical inventions, like creative histograms, representing the statistical data.

We know that the composer in the Middle Ages used mathematics like the Boethius mathematical means (arithmetic, geometric, and harmonic); and that composers in the twentieth century have and are using mathematical programs based on set-theory, probability, and fractals. Some have even written about their processes and tried to explain their mathematical applications; we also know about the fifteenth century Renaissance involvement with mathematics in sculpture, music, painting, architecture. It is the period of the classics and romantics in Europe, the seventeenth and eighteenth centuries (with the exception of J. S. Bach) which we mostly think devoid of any such measurements. As we will see in Franz Schubert and Frederic Chopin, composers of the eighteenth century, this is an erroneous assumption. They too arranged their musics with wondrous orders.

The models do not guarantee the eternal perfection of the composition, they just help us with another comprehension of the work, aiding us in delivering its overall structure and eventual performance. What the mathematical models provide is a world of common denominators for all musics from Africa, Europe, the Americas. When discussing the models we mention such developments as the notion of linearity and non-linearity, the latter exploding in the twentieth century. Music has tended to be continuous and when discontinuous most find it difficult to follow its deployment. The models also prove that there is no age for music's architecture -- an antiphon of Hildegard von Bingen shows similar measurements as does a work of the Hungarian contemporary Gyorgy Ligeti. These are the learnings that the mathematical models offer and which in the end are significant to erasing the fragmentation still existing amongst diverse musics, geographically and historically speaking.